The redistributive effects of bank capital regulation

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Abstract

We build a general equilibrium model of banks' optimal capital structure, where bankruptcy is costly and investors have heterogenous endowments and incur a cost for participating in equity markets. We show that banks raise both deposits and equity, and that investors are willing to hold equity only if adequately compensated. We then introduce (binding) capital requirements and show that: (i) it distorts investment away from productive projects toward storage; or (ii) it widens the spread between the returns to equity and to deposits. These results hold also when we extend the model to incorporate various rationales justifying capital regulation.

Keywords: limited market participation, bank capital structure, capital regulation, investor returns

JEL classifications: G21, G28

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1 Introduction

The regulation of financial institutions, and banks in particular, has been at the forefront of the policy debate for a number of years. Much of the concern relates to the perceived negative consequences associated with a bank's failure, and with how losses may be distributed across various stakeholders, such as borrowers (either corporate or individual) and creditors, including depositors and the government, with the ultimate bearers of the losses being households and shareholders.

A primary tool for bank regulation is the imposition of minimum capital standards, which amount to requirements that banks limit their leverage and issue at least a minimal amount of equity. Capital regulation has two primary roles. First, by creating a junior security held by bank shareholders, capital (i.e., equity) stands as a first line of defense against losses. Second, by forcing shareholders to have "skin in the game", capital helps control risk shifting problems that may arise as a result of investment decisions by levered banks. In fact, recent calls among regulators, policy makers, and academics (see, e.g., Admati et al., 2013) have been for banks to dramatically increase the amount of capital they issue as way of reducing risk and ultimately increasing social welfare.

What is less understood in the discussion related to bank capital regulation concerns its potential redistributive effects. If the primary goal of banks, or bankers, is to maximize profits, and capital structure is chosen taking this objective into account, then the imposition of a leverage constraint, or any other restriction on banking activities, will have an impact on bank profitability and consequently the return available to bank claimants. This raises the question how changes in bank profitability induced by binding capital regulation will be redistributed among the various claimants. This is an important consideration for understanding the incidence of regulation, particularly if part of the aim of regulation is to protect specific agents. Surprisingly, there has been little study on which party is mostly affected by changes to bank profitability induced by regulatory impositions.

To tackle these questions, we present a general equilibrium model of banks' optimal capital structures. Investors, who have heterogenous endowments can either store their wealth or invest in banks in the form of either equity or deposits. As is typical in the literature on stock market participation (see for example, Allen and Gale, 1994, Heaton and Lucas, 1996, Orosel, 1998, Polkovnichenko, 2004, Gomes and Michaelides, 2005, 2008, Alan, 2006, Chien et al., 2011, Favilukis, 2013, and Vissing-Jørgensen, 2003 for supporting empirical evidence), investors are reluctant to participate in financial markets and bear a cost to do so.

Banks exist to channel funds from investors into productive but risky investments and can finance themselves with either equity (i.e., capital) or deposits. Capital is valuable as it helps reduce default risk, making deposits safer and allowing the bank to reduce interest rates on deposits. Investors' decisions on holding bank equity capital or deposits is thus endogenous, and will depend on the difference in the returns to the two securities, which are themselves endogenous, reflecting the general equilibrium nature of the model.

As a first important step, we characterize the market equilibrium in a baseline model with no frictions. We show that banks always find it optimal to raise strictly positive amounts of both equity and deposits, with the optimal combination depending on the profitability of their investment projects. Banks' use of capital reduces the risk of bankruptcy, allowing the bank to raise deposits at a lower interest since investors will, ceteris paribus, prefer deposits that are safer, in line with the evidence in Afonso et al. (2011) and Pérignon et al. (2018) that depositors benefit from more capital as this protects them from the bank's default risk. The existence of participation costs leads to endogenous market segmentation, so that in equilibrium only investors with larger amounts of wealth hold equity, while those with lower amounts of wealth prefer to hold deposits or store. However, while the presence of participation costs endogenously segments the market, the size of these costs alone does not fully explain the difference between the expected return to shareholders and depositors.

We show that all equity investors, except for the marginal one, earn a strictly positive rent, even net of participation costs. The bank therefore creates value for investors by channeling funds from storage into real investment projects. For depositors, they earn a premium (over storage) when the expected return of bank investment projects is high and all funds flow to the banking sector. When project returns are low, however, not all funds are channeled to the banking sector and depositors' return is pinned down by the return to storage. We denote the former situation as *full inclusion* and the latter as *partial inclusion*.

We then turn to analyze the redistributive effects of capital regulation. We start with the frictionless baseline model, which yields no distortions, and identify two main implications associated with binding minimum capital requirements depending on how profitable are the investment projects. When project returns are relatively low and not all funds are being invested in productive projects, requiring banks to hold greater amounts of capital reduces the number of projects that are funded. This, in turn, reduces aggregate surplus since investment projects yield a higher surplus than investing in storage, thus lowering shareholders' returns. While bank default risk goes down, depositors are indifferent in equilibrium to the tighter regulation since their expected utility is pinned down by their outside option of storage.

By contrast, once project returns are high enough that all investment funds are allocated to productive projects, the only way to induce more investors, who are behaving optimally given prevailing returns, to hold equity is to make it more attractive relative to holding a deposit. As a result, the gap in the two securities' returns must widen in order for the market for equity to clear, with more investors finding it optimal to hold equity.

We then extend our model to consider various market failures that justify a need to regulate capital. Specifically, we consider three settings that are commonly discussed as giving rise to a need for regulation: externalities arising from "fire sales" of bank assets that occur when many banks fail at once; distortions introduced by deposit insurance, which push banks to rely excessively on deposits; and risk shifting problems for banks due to limited liability. We first show that, while the specific effects of regulation depend again on the profitability of the banks' investment projects, in all these cases capital regulation has a role in increasing efficiency and thus social welfare. We then show that, despite regulation reducing bankruptcy risk, its incidence falls in a similar way as before since satisfying the capital standard always implies having to induce relatively more investors to hold equity rather than deposits. When capital regulation solves a coordination problems and increases bank profitability, as in the case of fire sale externalities, shareholders always appropriate larger increases in returns than depositors. By contrast, when bank profitability is reduced by capital regulation, the difference in returns to shareholders and depositors decreases in the partial inclusion region, thus affecting shareholders mostly, while it increases in the full inclusion region where depositors' returns are reduced proportionally more than shareholders' returns.

The model has a number of empirical implications. The first implication concerns the allocation of funds in the market equilibrium. Attributing variations in project expected returns as resulting from changes in the business cycle, the analysis shows that the size of the intermediated sector and banks' capital structures vary with market conditions. In particular, when times are good all investors benefit from improved aggregate productivity as banks compete for both deposits and equity. In contrast, at a trough in the business cycle where productivity is low and investors resort to storage, any change in market conditions primarily affects equity returns and, through market clearing, the number of banks that operate. As the business cycles improves, more investors participate in the equity market as the relative return differential between equity and deposits increases, consistent with the findings in Lin (2017).

The second implication concerns the effects of regulation, both for the structure of the banking sector and investors' returns. The results show that when market conditions are such that not all funds are channeled through the banking sector, capital regulation tends to reduce the number of active banks, thus leading to lower investment in productive projects. Although this is not welfare reducing, the result is consistent with the findings in a number of papers that shocks to banks' capital (as those imposed by regulation) reduce lending (see, e.g., Kashyap et al., 2010, and more recently, Fraisse et al., 2017 and Gropp et al., 2018). The implications for investors' returns crucially depend on how regulation affects bank profitability. When some of the benefits of regulation are captured by the banking sector, our results are then consistent with the findings in Baker and Wurgler (2015) that capital regulation makes banks less risky while at the same time increasing their realized stock returns; and in Bouwman et al. (2017) that banks with high capital have higher average risk adjusted stock returns than low capital banks, especially during bad times.

Our work is related to various strands of literature. A sizable literature has considered the role of bank capital in partial equilibrium frameworks where the return to equity is exogenously given.[1] Most of these studies support the view that banks, if left unregulated, hold inefficiently low levels of capital because of market failures or the presence of externalities, which in turn renders them excessively prone to failure.[2] Thus, imposing minimum capital standards on banks should increase welfare, and much of the literature has focused on discussing the mechanisms through which capital standards act, or on estimating their welfare effects (Van den Heuvel, 2008). Far less explored, however, is the question of where the incidence of such regulatory intervention falls, for which deriving securities' returns as equilibrium variables is critical.

Our findings that higher capital standards may not be beneficial for depositors is related to the work by Besanko and Thakor (1992) and Repullo (2004). Both contributions discuss the consequences of tighter capital standards in a spatial model of imperfect bank competition where returns to bank equity are exogenously fixed. They find that deposit rates fall because capital regulation forces banks to substitute some deposits for equity, which in turn prompts lenders to compete less aggressively for depositors. Perhaps closer in spirit to our focus, recent work by Arping (2018) shows that capital regulation benefits depositors if banks can reduce the frictions investors face when trying to invest in assets that serve as stores of value. If banks have limited opportunities to make productive investments, any excess funds will need

[1] See e.g., Holmström and Tirole (1997); Hellmann, Murdock, and Stiglitz (2000); Morrison and White (2005); Dell'Ariccia and Marquez (2006); Allen, Carletti, and Marquez (2011); Mehran and Thakor (2011); see Thakor (2014) for a survey.

[2] Berger and Bouwman (2013) document empirically that well-capitalized banks are more likely to withstand a financial crisis.

to be invested in storage. As a result, banks may be reluctant to raise deposits since doing so, represents a wealth transfer from shareholders to depositors. Capital regulation that forces banks to raise deposits may thus benefit depositors by giving them access to storage options at lower cost, and may benefit shareholders to the extent that it solves market coordination problems. In contrast, in our model all markets are perfectly competitive and investors can always freely choose how to invest given prevailing interest rates and expected returns, all of which are obtained endogenously in general equilibrium. From this perspective, our model is closest to Allen, Carletti, and Marquez (2015), where limited participation in the equity market is the key friction, although it is exogenously fixed. In our model, the degree of participation is endogenous and is a function of the wedge in the expected return between equity and deposit markets.

Our finding that depositors are often not the primary beneficiaries of capital regulation is reminiscent of studies that have investigated the incidence of taxation. For example, Huizinga, Voget, and Wagner (2014) have documented empirically that international corporate income taxation of banks is reflected in higher pre-tax interest rate margins, suggesting that the incidence of taxation falls primarily on bank customers and depositors rather than on shareholders.

By focusing on the endogenous degree of participation in the bank equity market, our paper is also related to the literature analyzing how investors' limited participation in financial markets has implications for the equilibrium pricing of assets. For instance, Allen and Gale (1994) study how limited market participation can lead to amplified volatility of asset prices. Vissing-Jørgensen (2002) uses limited market participation to help explain part of the equity premium puzzle. Several studies in the household finance literature also feature an endogenous degree of household participation in equity markets which arises due to heterogeneous household characteristics. For instance, Lusardi, Michaud, and Mitchell (2017) show in a lifecycle model that the heterogeneous cost of acquiring financial knowledge limits stock market participation of less wealthy households and can account for a significant portion of wealth inequality. Our paper contributes to this literature by showing that investors' reluctance to hold risky assets not only affects household wealth, asset allocations, or stock volatility, but it also has implications for the capital structures of financial firms.

The paper proceeds as follows. The next section lays out the model. Section 3 contains the characterization of equilibrium. Section 4 looks at social welfare and studies the effects of a binding capital requirement in the frictionless baseline model. Section 5 introduces various frictions that provide justifications for capital regulation. Section 6 discusses the cases of endogenous returns for both storage and the investment technology. Section 7 discusses the case where existing banks are forced to change their capital structures as part of a recapitalization. Finally, Section 8 concludes. All proofs are relegated in the Appendix.

2 A frictionless benchmark

We develop a simple one period ($t = 0, 1$) benchmark model of financial intermediation and investors that can provide funds in the form of equity capital or deposits. There exist two investment options: one is a *storage* technology, which yields in $t = 1$ a return of one on every unit of funds invested at $t = 0$; the other is a *risky* investment, which, for every unit of funds invested at $t = 0$, yields in $t = 1$ a risky return of $r = Rx$, where R is a positive constant and x is a random variable distributed over the $[0, X)$ interval according to a distribution function $F(\cdot)$. We assume $F(\cdot)$ to be differentiable and satisfy the increasing hazard ratio property, i.e., $\frac{F'(x)}{1-F(x)}$ is strictly increasing in x. This property is satisfied by many frequently used distributions, e.g., the uniform distribution, normal, etc.. We normalize x to have an expected value of one, and refer to the induced distribution of r as $G_R(\cdot)$.[3] With this normalization, the risky technology yields an expected return of $E[r] = \int_0^{RX} r\, dG_R(r) = R > 1$.

There is a continuum of risk-neutral investors, who have the choice of investing directly in the storage technology or placing their wealth in a bank, as either depositors or equity holders. Specifically, each investor has an endowment $w \in [\underline{w}, \overline{w}]$, which is drawn i.i.d. across

[3] Since $G_R(r) \equiv F(r/R)$, $G_R(\cdot)$ satisfies the increasing hazard ratio property whenever $F(\cdot)$ satisfies it.

investors from a continuous distribution $H(w)$. The aggregate wealth of all investors is M, where $M \equiv \int_{\underline{w}}^{\overline{w}} w\, dH(w)$. Each investor incurs a *participation* cost $c > 0$ to invest in bank equity, but bears no such cost if she holds only either bank deposits or storage. This cost, which, along with investor heterogeneity, is a standard feature in much of the literature on stock market participation (e.g., Allen and Gale, 1994, Heaton and Lucas, 1996, Orosel, 1998, Polkovnichenko, 2004, Gomes and Michaelides, 2005, 2008, Alan, 2006, or Chien et al., 2011) and finds empirical support in Vissing-Jørgensen (2003), can be thought as representing the resources needed to understand the basic features of the market, monitoring, transaction costs, or brokerage fees, but can also be thought as a disutility associated with other frictions common to banking markets such as adverse selection.[4]

Banks are primarily vehicles that provide investors with access to the risky technology. Each bank finances itself with an amount of capital k and an amount of (uninsured) deposits $1-k$ and invests in the risky technology.[5] This implies that, by becoming shareholders in a bank, investors de facto take a position in the risky technology. We denote the promised per unit deposit rate as r_D, and the equilibrium expected return to bank deposits and to bank capital as u and ρ, respectively.

Banks are subject to bankruptcy if they are unable to repay their debt obligations. This occurs when $r < (1-k)r_D$, that is, when the realized return from the risky technology is lower than the total promised repayment to depositors. Bankruptcy is costly, and for simplicity we assume that in the event of bankruptcy, all the project's return is dissipated.[6] Finally, we consider the banking sector as being perfectly competitive. Free entry reduces excess returns to zero, and banks behave as price takers with respect to the expected return on bank capital ρ and deposits u.

[4]Irrespective of the specific justification for the existence of participation costs, the literature still recognizes them as being likely responsible for limited participation in the stock market, although such costs seem to have declined over time (see, e.g., the discussion in Favilukis (2013)). In line with this, it is worth noting that our results continue to hold even if the participation cost c is vanishingly small.

[5]Given there are constant returns to scale, normalizing the size of every bank to 1 is without loss of generality.

[6]We relax this assumption later in Section 5.1.

3 Optimal capital structure

The equilibrium of the model consists of a vector $\{k^*, r_D^*, \rho^*, u^*,\ N^*\}$ representing bank capital structure and the promised deposit rate, investors' expected returns on bank capital and deposits, and the number of active banks such that:

1. Investors optimally decide whether to invest in deposits, equity, or storage so as to maximize their expected utility;
2. Banks choose capital k and the deposit rate r_D to maximize excess returns;
3. Free entry reduces bank excess returns to zero in equilibrium;
4. The markets for bank equity and deposits clear.

We start by analyzing investors' optimal investment strategy for given expected returns ρ and u. An investor of wealth w choosing to hold equity obtains a return ρ on her investment but incurs a participation cost of c such that her overall payoff is $\rho w - c$. If the same investor chooses to hold deposits instead, there is no participation cost and her payoff is uw. Finally, investing in storage yields a payoff of w. Thus, an investor is willing to hold deposits over storage if $uw \geq w$ and thus if $u \geq 1$. Similarly, an investor is willing to hold equity over deposits and storage if $\rho w - c \geq uw \geq w$. It follows that whenever $\underline{w} \leq \frac{c}{\rho - u} \leq \overline{w}$ there exists a marginal investor with wealth $\widehat{w} = \frac{c}{\rho - u}$, who is indifferent between equity and deposits.

We can now calculate the total investor demand for bank equity, K, as follows. For a given spread $\rho - u$ with $u \geq 1$, the aggregate demand for bank equity is

$$K = \begin{cases} 0 & \text{if } \rho - u \leq c/\overline{w} \\ \int_{\frac{c}{\rho-u}}^{\overline{w}} w\, dH(w) & \text{if } c/\overline{w} < \rho - u < c/\underline{w} \\ M & \text{if } \rho - u \geq c/\underline{w} \end{cases} \tag{1}$$

Similarly, the aggregate demand for deposits, D, is equal to 0 if $u < 1$, and to $\int_{\underline{w}}^{\frac{c}{\rho-u}} w\, dH(w)$ if $u > 1$, while investors are indifferent between holding deposits or storage when $u = 1$. We then denote as $D + S = \int_{\underline{w}}^{\frac{c}{\rho-u}} w\, dH(w)$ the total demand for deposits and storage, where S is the aggregate holding of storage.

Turning now to the banks' problem, each bank chooses its capital k and its promised deposit rate r_D to solve:

$$\max_{k,r_D} E\left[\Pi_B\right] = \int_{r_D(1-k)}^{RX} (r - (r_D(1-k))dG_R\left(r\right) - \rho k \tag{2}$$

subject to

$$E\left[U_D\right] = \int_{r_D(1-k)}^{RX} r_D dG_R\left(r\right) \geq u \tag{3}$$

$$E\left[\Pi_B\right] \geq 0 \tag{4}$$

$$0 \leq k \leq 1. \tag{5}$$

Expression (2) represents the excess return of the bank. The first term captures the bank's expected return from investing in the risky technology, net of the payment $r_D(1-k)$ to depositors. Such a return is positive only when the bank does not go bankrupt, that is for $r \geq r_D(1-k)$. The second term ρk reflects the expected return to shareholders for providing capital (i.e., equity) to the bank. Constraint (3) captures depositors' participation constraint. It requires that the payoff depositors receive when the bank remains solvent is at least equal to their opportunity cost u. Constraints (4) and (5) ensure that the bank is active and that the chosen capital structure lies within the feasible range.

In equilibrium both the equity market and the market for deposits have to clear. Aggregating the individual choices of capital and deposits of all banks gives the aggregate supply of equity and deposits. Denoting as N the number of banks, market clearing then requires $Nk = K$, so that the supply of bank capital, Nk, equals demand from (1), and $N(1-k) = D$

so that the supply of deposits equals demand.

We can now characterize the equilibrium.

Proposition 1. *The model has a unique equilibrium where* $k^* \in (0,1)$*,* $r_D^* \in (u^*, RX)$*, and* $\rho^* > E[r] > u^*$*. Moreover, for some threshold value* $\overline{R} > 1$*, the following holds:*

1. *For* $E[r] < \overline{R}$*, there is* partial inclusion: $N^* < M$ *and* $u^* = 1$*.*

2. *For* $E[r] \geq \overline{R}$*, there is* full inclusion: $N^* = M$ *and* $u^* \geq 1$*.*

The proposition shows that banks always find it optimal to raise a combination of equity and deposits, each of them in strictly positive amounts. The choice between equity and deposits entails a trade-off. On the one hand, capital is valuable as it helps reduce default risk and thus bankruptcy costs. All things equal, this makes deposits issued by the bank safer and thus makes it easier to attract deposits, allowing the bank to reduce r_D, consistent with the findings in Afonso et al. (2011) and Pérignon et al. (2018). However, greater amounts of capital require that the bank offers a greater return to investors in order to compensate them for incurring the costs of participation. The equilibrium capital structure balances these two forces: the optimal amount k^* leaves the bank exposed to default risk, while at the same time generating enough surplus to allow a greater premium to be paid to investors willing to become equity holders.

The equilibrium return ρ^* for equity holders is always greater than the expected project return, $E[r]$, while that for depositors u^* is lower. This has important implications for the distribution of returns across investors. The existence of a participation cost c leads to an endogenous market segmentation, whereby in equilibrium only investors with large amounts of wealth $w \geq \widehat{w}$ hold equity while those with less wealth ($w < \widehat{w}$) will either hold deposits or invest in storage. Thus, all equity investors (except for the marginal one with wealth $w = \widehat{w}$) earn a strictly positive premium per dollar invested over what they could get from holding their wealth in bank deposits, even net of the participation cost c given that $w\rho^* - c > wu^*$ for any $w > \widehat{w}$.

Finally, the proposition shows that the size of the banking sector as well as depositors' return depend on project expected returns. When this is low (i.e., for $E[r] \leq \overline{R}$), the amount of investors' funds allocated to the banking sector, as measured by the number of banks N^*, is less than the total funds available M so that some investors choose to keep their funds in storage. In this region, which we refer to as *partial inclusion*, the equilibrium return to depositors u^* is pinned down by the return to storage, which is equal to 1. For higher levels of the project's expected return (i.e., for $E[r] \geq \overline{R}$), the investment projects are sufficiently profitable that the equilibrium allocation has all funds flowing to the banking sector, $N^* = M$, and all investors either hold deposits or become equity holders. In this region, which we will refer to as *full inclusion*, the equilibrium return to depositors is no longer driven by the return to storage so that $u^* > 1$.

We next conduct some comparative statics. We start with the profitability of the bank's projects.

Corollary 1.1. *An increase in the project's expected return, $E[r] = R$, leads to:*

i) a decrease in k^* for $E[r] < \overline{R}$ and an increase thereafter;

ii) an increase in the probability of bankruptcy for $E[r] < \overline{R}$ and a decrease thereafter;

iii) an increase in ρ^* for any $E[r]$ and an increase in u^* for $E[r] \geq \overline{R}$;

iv) an increase in N^* for $E[r] < \overline{R}$;

v) an increase in total capital as measured by $K^* = N^* k^*$ for any $E[r]$.

The corollary shows that the capital structure k^* at the individual bank level exhibits a non-monotonic dependence on project expected return. To see why, consider first the case where $E[r] < \overline{R}$ so that returns to deposits are unaffected by an increase in project expected return. The increased project profitability of projects will then lead to an increase in ρ, drawing more equity capital into the banking sector. But as ρ increases, deposit funding becomes relatively more attractive for the bank, leading each bank to prefer to be less capitalized and rely instead on deposits. The market for capital clears by inducing greater entry by banks and thus a greater degree of financial inclusion, with each bank having less capital as $E[r]$

increases.

In contrast, in the region of full inclusion, for $E[r] \geq \overline{R}$, the equilibrium return to deposits increases as project returns increase since the banks, which are now fixed in number, compete more aggressively for deposits. This general equilibrium feedback to deposit returns reduces the benefit of higher leverage, and at the same time increases the expected bankruptcy costs since the deposit rate must rise. As a consequence, banks reduce their leverage once the point of full inclusion is reached. This leads to higher returns for both equity and deposit holders as well as to an increase in the total amount of capital employed in the banking sector.

While the model is static, the variable R can be viewed as capturing changes in the business cycle, where high values of R represent market booms in the business cycle, and low values are troughs. From that perspective, Proposition 1 along with Corollary 1.1 suggest that as market conditions improve, more investment funds will flow into the productive banking sector, and will exit when conditions worsen. These movements in aggregate productivity affect equilibrium returns to bank securities. Specifically, when times are good all investors benefit from improved aggregate productivity as banks compete for both deposits and equity, while as times worsen and storage returns come into use, any change in market conditions primarily affect equity returns and, through market clearing, the number of banks that operate. Either way, as the business cycles improves, the relative return differential between equity and deposits increases as more investors participate in the equity market, consistent with the findings in Lin (2017).

We now turn to analyze how the equilibrium changes with investors' participation cost.

Corollary 1.2. *An increase in the participation cost c leads to: (i) for $E[r] < \overline{R}$, a decrease in N^*, while leaving k^*, u^*, or ρ^* unchanged; (ii) for $E[r] \geq \overline{R}$, a reduction in k^* and u^*, and an increase in ρ^*. The threshold for full inclusion, $\overline{R}$, is increasing in c.*

The corollary shows that while participation costs lead to an endogenous market segmentation, these costs do not fully explain the size of the difference between the expected return to shareholders and depositors. An increase in c always makes investors more reluctant to participate in equity markets through a reduction in the demand for equity as given in (1). When, for $E[r] < \overline{R}$, some investors choose to hold their wealth outside of the banking sector (i.e., $N^* < M$), increases in the cost of participating have *no effect* on the equilibrium expected returns. The reason is that the lower demand for equity is compensated through a reduction in the number of active banks, N^*, rather than through changes in ρ^* or u^*. By contrast, once $E[r] \geq \overline{R}$ and $N^* = M$, the only way to restore equilibrium when an increase in c makes investors more reluctant to participate in equity markets is for prices to adjust, so that the difference $\rho - u$ must increase.

4 The incidence of binding capital requirements

We now turn to the question of how capital requirements affect banks and, by extension, investors funding them. To study this issue, we first establish that our benchmark model provides a market solution that is coincident with what a central planner would choose, so that the banking sector fully internalizes the (social) benefit of holding capital as well as investors' costs of participating in equity markets. It follows that any additional capital requirement in this framework is distortionary. Yet, analyzing its effects on investors' returns helps understand who primarily bears the burden of regulation. We will consider the possible benefits of regulation in Section 5.

Consider the case where a central planner chooses bank capital to maximize social welfare (i.e., investors' aggregate returns net of aggregate participation costs), while the deposit rate is set by the bank in order to maximize its expected excess return. This means that the planner solves

$$\max_{k} SW = \rho K + uD + (M - N) - \int_{\widehat{w}}^{\overline{w}} c dH(w) \tag{6}$$

subject to

$$r_D = \arg\max_{r_D} \int_{r_D(1-k)}^{RX} (r - (r_D(1-k)) dG_R(r) - \rho k \tag{7}$$

and (3), (4), and (5). The constraints are as in the market problem except (7), which indicates that the deposit rate r_D is chosen by the bank to maximize its excess returns. We can now state the following result.

Proposition 2. *The central planner's allocation of bank capital coincides with the competitive capital structure in Proposition 1.*

Although banks behave as price takers in the competitive equilibrium and do not individually consider the impact of their capital structure choices on the equilibrium rate of return on capital, they issue the same amount of bank capital as what a central planner would choose. The reason is that there are no externalities in the bank equity market in our model and no social losses associated with bank default beyond the dissipation of project returns. Banks maximize returns to the benefit of bank shareholders and, given the market for capital is competitive, they ultimately internalize investors' costs of participating in equity markets. It bears noting as well that Proposition 2 does not say that there is no social value to bank capital, but rather that the social and the private value of capital coincide in our benchmark model.

We next study the issue of the incidence of binding capital requirements, that is $k^{reg} > k^*$, where k^* denotes the market solution for capital from Proposition 1. Denoting K^* and K^{reg} as the aggregate amount of capital in the market and regulatory solutions, and $N^*$and N^{reg} as the corresponding number of banks, we have the following.

Proposition 3. *Suppose that $k^{reg} > k^*$ and:*

1. *$N^* < M$, with $u^* = 1$. Then, $u^{reg} = 1$, $\rho^{reg} < \rho^*$, and $N^{reg} < N^*$, so that $K^{reg} < K^*$ as well.*

2. *$N^* = M$, with $u^* > 1$. Then, $\rho^{reg} - u^{reg} > \rho^* - u^*$ and $u^{reg} < u^*$, so that the difference in returns to shareholders and depositors increases, and the return to depositors decreases.*

The proposition establishes that binding capital requirements lead to fewer banks operating in the case of partial inclusion, when $N^* < M$. Since productive projects are funded through banks, the induced reduction in the number of active banks in the economy leads to an inefficiency in terms of lower output being produced.

The proposition also highlights how binding capital requirements affect the returns of the different classes of investors. Since the market solution maximizes aggregate output, a binding capital requirement always leads to less total surplus. In the partial inclusion region, the lower output translates into a lower return to equity holders. The reduction in the return to shareholders, together with the lower amount of capital in use (i.e., $K^{reg} < K^*$), means that $\widehat{w}^{reg} > \widehat{w}^*$, where $\widehat{w}^{reg}$ and $\widehat{w}^*$ represent the marginal investors in the regulatory and market solutions, respectively. It follows that individuals with $\widehat{w}^* \leq w < \widehat{w}^{reg}$, who in the market solution would have held equity, now hold deposits instead. Clearly, these switching investors are worse off than in the market solution since they now obtain a return of 1 rather than $\rho^* > 1$. However, holding deposits is now optimal for them as $\rho^{reg} w - c < w$.

In the full inclusion region, since the number of projects that are financed remains constant when banks are required to increase capital (for local changes in the amount of capital around the market solution k^*), capital requirements entail a deadweight loss in terms of increased participation costs borne by the additional investors that need to be induced to hold bank equity. While equilibrium returns to all investors decrease, the return u^* to depositors decreases even more because the difference $\rho^* - u^*$ must increase in order for the market to clear with more investors willing to hold bank capital. Thus, while shareholders earn a lower return, reflecting the greater deadweight losses and lower aggregate output, depositors see their return decrease even more. In this sense, depositors suffer a more than commensurate reduction in the return they earn in equilibrium. As a consequence, capital regulation may be seen as a channel to widen the return differentials between sophisticated and unsophisticated investors, thus amplifying income inequality, as argued in the recent literature on limited market participation and household wealth accumulation (e.g., Lusardi et al., 2017).

Given that the number of banks remain constant in the full inclusion region, in equilibrium there must be an increased use of capital in aggregate (i.e., $K^{reg} > K^*$) and thus $\widehat{w}^{reg} < \widehat{w}^*$ must hold. This means that individuals with $\widehat{w}^{reg} \leq w < \widehat{w}^*$, who in the market solution would have held deposits, now hold equity instead. Since $u^{reg} < u^*$, the switching investors will be now better off (net of participation cost) only if $\rho^{reg} w - c > u^* w$.

5 Market frictions and bank capital regulation

The previous sections have presented a frictionless baseline model, where banks' capital structure decisions are constrained efficient and there is no scope for capital regulation. Now we extend the model to incorporate various frictions so that the social and private value of bank capital no longer coincide and capital regulation can increase social welfare.

Specifically, we study some canonical market failures associated with financial intermediaries. The first is the presence of externalities in the recovery value of assets that may arise when many banks fail at once – "fire sales" – and that may depress asset values. In this case, capital regulation solves a coordination problem for banks and as a result benefits the banking sector. The second market failure derives from the introduction of deposit insurance, which provides an implicit or explicit subsidy for raising deposits rather than equity, and tilts banks' capital structures toward being excessively levered. Here, regulation improves social welfare but acts as a tax on the banking sector since it reduces the extent to which banks can take advantage of the public safety net. Finally, we consider a risk shifting problem induced by limited liability, which leads banks to take excessive risk. Here regulation again increases social welfare, but has an ambiguous effect on bank profitability.

For simplicity, in what follows we assume that the distribution function $G_R(\cdot)$ for project returns is a uniform distribution in $[0, 2R]$.

5.1 Fire sale externalities

So far we have assumed that in the case of bankruptcy the entire project return is dissipated. Consider now a modification where liquidation yields a recovery value equal to a fraction $h < 1$ of the realized cash flow r and that such a value depends on how many other banks are in default and thus being liquidated. In other words, losses under bankruptcy are equal to $(1-h)r$, where h decreases in the number of active banks N. This captures the idea that the failure of many banks at once depresses asset prices for all banks that are being liquidated – a "fire sale" externality.

As before, each bank chooses the amount of capital k that maximizes its expected excess returns, as given by (2). The only change to the bank's problem stems from depositors' participation constraint, which now incorporates that depositors may receive something in the event of bankruptcy, and is given by

$$E\left[U_D\right] = \frac{1}{2R}\int_0^{r_D(1-k)} \frac{hr}{1-k}dr + \frac{1}{2R}\int_{r_D(1-k)}^{2R} r_D dr \geq u. \tag{8}$$

The recovery under default is reflected in the first term, $\frac{hr}{1-k}$, while the second term is the promised repayment, which is made when the bank is solvent, $r \geq r_D(1-k)$.

Banks choose their capital structure disregarding the effect of their choice on the equilibrium asset liquidation value. By contrast, a central planner would choose the amount of capital at each individual bank to maximize total surplus as given by

$$SW = N\frac{1}{2R}\int_0^{r_D(1-k)} hr dr + N\frac{1}{2R}\int_{r_D(1-k)}^{2R} r dr + (M-N) - \int_{\widehat{w}}^{\overline{w}} c dH(w). \tag{9}$$

The first two terms represents the surplus produced when banks go bankrupt and generate the recovery value hr, and when they remain solvent and produce the project return r. The third term is the return from any potential investment in storage, while the last term captures the participation costs of all investors with wealth $w \geq \widehat{w}$.

As usual, denote by k^* and k^{reg} the equilibrium capital structures in the decentralized and the central planner's problems, respectively, and by N^* and N^{reg} the number of banks in the respective cases. We then have the following result.

Proposition 4. *In the case of fire sale externalities, we have $k^{reg} \geq k^*$ and $N^{reg} \leq N^*$, with the inequalities strict whenever $N^{reg} < M$.*

The proposition establishes that there is a social value to requiring banks to hold more capital than what they are inclined to do as a way of reducing the externalities associated with fire sales in asset prices. By requiring banks to hold more capital, not only do banks face lower bankruptcy costs but, more importantly, the central planner succeeds in reducing the number of banks that will operate and, hence, possibly go bankrupt. The contraction in the number of banks and thus, ultimately, in aggregate lending is consistent with the findings in a number of papers examining the effects of shocks to banks' capital, such as those imposed by regulation, on lending (e.g., Kashyap et al., 2010, Fraisse et al., 2017, and Gropp et al., 2018 [CUT, and various papers cited therein]). However, the reduction in lending has a social value in our context as it reduces bankruptcy costs through greater recovery values (i.e., higher h).

We now turn to the question of how the increased surplus is allocated among investors. Define ρ^* and ρ^{reg} as the return to shareholders and u^* and u^{reg} as the return to depositors in the market and in the regulatory solutions, respectively. We have the following.

Corollary 4.1. *When there are "fire sale" externalities, we have $\rho^{reg} > \rho^*$, $u^{reg} = u^* = 1$ and $K^{reg} > K^*$for $N^{reg} < M$.*

The result establishes that, while capital regulation has value in that deposits become safer as a result of the binding capital requirements, shareholders are the primary beneficiaries of the increased surplus that is generated. Capital regulation increases total surplus, which must be allocated between investors that hold deposits and those that hold equity. The increased amount of capital at each bank reduces bankruptcy risk and hence, all things equal, increases the return to depositors. This allows banks to reduce the deposit rate, so that in equilibrium depositors can continue to earn a return equal to their opportunity cost of holding deposits. This implies that, when $N^{reg} < M$, the increased surplus accrues to shareholders even if leverage, $1 - k$, decreases. Thus, ρ goes up, with a greater aggregate amount of equity being employed in the banking sector, despite the reduction in the number of banks. As a consequence, $\widehat{w}^{reg} < \widehat{w}^{*}$, so that individuals with $\widehat{w}^{reg} \leq w < \widehat{w}^{*}$ find it optimal to switch from holding deposits to holding equity. In contrast to the baseline model, the switching investors are now better off given that $\rho^{reg}w - c > u^{reg}w = u^{*}w = w$.

Once $N^{reg} = M$, further increases in capital requirements no longer affect the number of banks that operate and can possibly fail, and so there is no further role for binding capital requirements. As a result, in that region the planner's solution coincides with the market solution.

To sum up, the presence of fire sale externalities that may arise when banks are liquidated provides a rationale for the introduction of capital requirements. The increased social surplus banks generate leads to a greater use of capital in the banking system, although also to fewer active banks, and to higher expected returns to shareholders. This latter result is consistent with some recent empirical findings. For example, Baker and Wurgler (2015) find that capital regulation makes banks less risky, while at the same time increasing their realized stock returns and thus shareholders' returns. Similarly, Bouwman et al. (2017) find that banks with high capital have higher average risk-adjusted stock returns than low capital banks, especially during bad times. This suggests that, as in our model, capital, and thus capital regulation, plays a special role in reducing bankruptcy risk and thus determining stock returns.

5.2 Deposit insurance

In the analysis above we assumed that deposits are not insured, so that the interest rate on deposits fully reflects the bank's default risk. Suppose now that deposits are insured, at least partially. As has often been argued (see, e.g., Boot and Greenbaum, 1993, and Demirgüç-

Kunt and Detragiache, 2002), deposit insurance encourages excessive risk taking by reducing banks' incentives to raise capital, thus providing a motivation for capital regulation.

To incorporate deposit insurance, assume that a fraction γ of deposits will be repaid by the deposit insurance fund upon bank failure. No insurance corresponds to the case where $\gamma = 0$, while full insurance corresponds to the case where $\gamma = 1$. For simplicity, we also assume that the insurance is financed from the proceeds of non-distortionary lump sum taxes.[7] In this case, the bank chooses capital k and r_D so as to maximize

$$\max_{k,r_D} \frac{1}{2R} \int_{r_D(1-k)}^{2R} (r - r_D(1-k))\, dr - \rho k \tag{10}$$

subject to

$$E[U] = \frac{1}{2R} \int_0^{r_D(1-k)} \gamma r_D dr + \frac{1}{2R} \int_{r_D(1-k)}^{2R} r_D dr \geq u, \tag{11}$$

where the first term represents the provision of deposit insurance when the bank is in default and each depositor receives a fraction γ of the promised payment r_D, while the second term is the repayment in case of bank solvency.

The provision of insurance distorts banks' capital structure decisions, as shown in the following result, where we indicate with the superscript γ the market solution in the case of deposit insurance and with $*$ the solution in the market equilibrium of Section 3.

Proposition 5. *In the presence of deposit insurance of amount $\gamma > 0$, the unique equilibrium is characterized by $k^\gamma < k^*$. As γ increases, banks optimally choose to hold less capital: $\frac{dk^\gamma}{d\gamma} < 0$.*

In equilibrium banks hold less capital than when deposits are uninsured, thus operating with more leverage. This occurs because the rate on deposits is less sensitive to the probability of bankruptcy and thus depends less on the amount of leverage banks choose. It follows that banks have less incentives to raise capital as a way of reducing bankruptcy risk since their

[7] The results continue to hold if deposit insurance is priced such that it is actuarially fair from an ex-post perspective (see Allen, Carletti, and Marquez, 2015).

cost of borrowing (i.e., deposit rates) does not fully reflect this reduction in risk.

We next show that banks' choices will be socially suboptimal in comparison to the capital structure chosen by a central planner whose objective is to maximize social welfare, as given by

$$SW = N\frac{1}{2R}\int_{r_D(1-k)}^{2R} rdr + (M-N) - N\frac{1}{2R}\int_{0}^{r_D(1-k)} \gamma r_D (1-k)\, dr - \int_{\widehat{w}}^{\overline{w}} cdH(w), \quad (12)$$

subject to the usual constraints. The primary difference between the planner's objective function and that of the banks is that the planner internalizes the cost of providing deposit insurance, as represented by the third term in the above expression. Denote as k^{reg} the solution to the planner's problem in (12). We obtain the following.

Proposition 6. *For any $\gamma > 0$, so that deposits are insured, we have $k^{reg} > k^{\gamma}$. For $R \leq \overline{R}$, capital regulation reduces ρ and N (i.e., $\rho^{reg} < \rho^{\gamma}$ and $N^{reg} < N^{\gamma}$) , while for $R > \overline{R}$, it widens the gap between shareholder and depositor returns (i.e., $\rho^{reg} - u^{reg} > \rho^{\gamma} - u^{\gamma}$).*

The proposition shows that the optimal regulatory solution entails banks holding more capital than what they would individually find optimal, and it has the usual implications for investors' returns and on number of banks in the partial inclusion region as in the baseline model.

Although capital regulation improves welfare relative to an economy with deposit insurance only and no regulation, the presence of deposit insurance is by itself inefficient. In other words, there is no justification for insuring depositors in our framework given that, as shown in Section 4, the unregulated equilibrium without deposit insurance yields the same allocation as would be chosen by a central planner. To justify the role of deposit insurance, we next show that the combination of capital regulation and deposit insurance improves upon the completely unregulated case (i.e., when there is neither deposit insurance nor capital regulation).

Corollary 6.1. *Total surplus in an economy with deposit insurance $\gamma > 0$ and (optimal) capital regulation is strictly higher than in the absence of either of them: $SW^{reg} > SW^{*}$.*

The corollary establishes that deposit insurance, coupled with minimum capital standards, increases surplus overall relative to the case where the market is entirely unregulated and there are no guarantees on bank liabilities. For the case where $N^{reg} < M$, this translates into increases in the expected return to shareholders (i.e., $\rho^{reg} > \rho^{*}$), while when $N^{reg} = M$, both depositors' and shareholders' expected returns may increase as a result of capital regulation when deposits are insured relative to the market solution.

5.3 Risk shifting induced by limited liability

The final friction we study stems from the possibility of risk shifting – that shareholders take actions to increase project risk since they view debtholders/depositors as bearing most of the increase in possible losses, while reaping the bulk of the benefits of any greater upside. This type of agency problem is often identified as one of the larger problems for financial institutions due to their greater leverage relative to non-financial firms. Moreover, the possibility of risk shifting creates a direct conflict between bank shareholders and depositors. As we show below, regulation in this case encompasses the two forces identified in the previous two sections: on the one hand, it benefits the banking sector directly while, on the other hand, it represents a tax. As a consequence, whether on net banks benefit or have reduced profitability as a result of regulation depends on the relative magnitudes of the effects.

To study such a setting, we modify the model slightly to allow the bank or, equivalently, bank shareholders to take a privately costly action a aimed at increasing the variance of project returns without affecting the mean. In other words, it is a mean-preserving spread (MPS) of project return r. Specifically, by risk shifting an amount a at cost $\frac{a^2}{2}$ at time $t = \frac{1}{2}$, the probability of each extreme outcome becomes $\Pr(r = 0) = \Pr(r = 2R) = \frac{a}{2}$ and, as a consequence, the density in the interior $r \in (0, 2R)$ becomes $\frac{1-a}{2R}$. The rest of the model and timing are as before, with the bank choosing k and r_D at $t = 0$, and then output is realized at $t = 1$.

As in Section 5.2, we assume that there is (partial) deposit insurance that covers a fraction $\gamma \in (0, 1)$ of each deposit. This means that, for a given level of capital k and deposit rate r_D, a depositor's expected utility is given by

$$\frac{a^C}{2}\gamma r_D + \frac{1-a^C}{2R}\int_0^{r_D(1-k)} \gamma r_D dr + \frac{1-a^C}{2R}\int_{r_D(1-k)}^{2R} r_D dr + \frac{a^C}{2} r_D, \tag{13}$$

where a^C is depositors' conjecture concerning the amount of risk shifting the bank will do. The first two terms represent the expected returns to depositors when the bank defaults (i.e., with probability $\frac{a^C}{2}$ when the realized project return is $r = 0$ and for $r \in (0, r_D(1-k)))$ and the insurance covers a fraction γ of the promised payment r_D. The last term is just the repayment a depositor gets in non-default states for $r > (0, r_D(1-k))$. As before, deposit insurance will lead the bank to choose an inefficiently low level of capital, which will then have important implications for risk shifting by the bank.

In choosing its degree of risk shifting, the bank maximizes

$$\max_a \Pi_B = \frac{1-a}{2R}\int_{r_D(1-k)}^{2R} (r - r_D(1-k))\, dr + \frac{a}{2}(2R - r_D(1-k)) - \frac{a^2}{2} \tag{14}$$

at $t = 1/2$, for given k and r_D.

Lemma 7. *The profit maximizing degree of risk shifting, a^*, is decreasing in the amount of capital k.*

The lemma formalizes a standard result related to risk shifting, which is that its extent depends on the degree of leverage for the bank. The lower is leverage, or equivalently, the higher is bank capital, the less risk shifting will banks find optimal to do. Indeed, at the limit as $k \to 1$, banks optimally would choose $a^* = 0$ since the bank would fully internalize the impact of its actions. At $t = 0$, the bank chooses its capital k and the deposit rate r_D so as to maximize (14).

Consider now the case where capital at each individual bank is chosen by a central planner that maximizes total surplus as given by

$$SW = N\frac{1-a}{2R}\int_{r_D(1-k)}^{2R} r\,dr + N\frac{a}{2}2R - N\frac{a}{2}\gamma r_D\left(1-k\right) \tag{15}$$

$$- N\frac{1-a}{2R}\int_{0}^{r_D(1-k)} \gamma r_D\left(1-k\right)\,dr - \int_{\widetilde{w}}^{\overline{w}} c dH\left(c\right) + M - N - \frac{a^2}{2}, \tag{16}$$

where both a and r_D are chosen by the banks. The expression is similar to before, with the third and fourth terms representing the provision of deposit insurance in default states of the world, which banks do not internalize. We have the following result, where we define k^* and k^{reg} as the market and the central planner solutions, respectively.

Proposition 8. *For the case of a bank risk shifting problem with deposit insurance $\gamma > 0$, we have $k^{reg} > k^*$, and $SW^{reg} > SW^*$. For $N^{reg} = M$, capital regulation as usual widens the gap between shareholder and depositor returns, $\rho - u$.*

The proposition shows that the central planner prefers a higher level of capital at each bank than in the market solution for two reasons: it reduces the inefficiency in the capital structure generated by deposit insurance and it attenuates the risk shifting problem. As always, capital reduces a bank's probability of default, lowering the expected social cost of bankruptcy. As in the previous section, the bank does not fully internalize this benefit because of the presence of deposit insurance. To the extent that capital regulation helps solve this distortion, it acts like a tax on banks. Additionally, the accompanying reduction in risk shifting further reduces default risk, lowering the social cost yet further. Risk shifting, however, represents a commitment problem on the side of the bank, which is exacerbated by the fact that deposits are insured. By attenuating this problem, capital regulation helps banks commit to shift risk less, and can increase profitability ex ante. Hence, whether banks themselves benefit or lose from regulation depends on which effect dominates.

As usual, in the region where $N^{reg} < M$, depositor returns are equal to their outside option, while shareholders may benefit to the extent that capital regulation raises bank profits and allows for greater distributions. Also as before, when $N^{reg} = M$, any capital requirement must lead to a widening of the difference $\rho - u$ in order to induce more investors to hold equity investments, with the net changes in ρ and u individually depending on whether bank output ultimately increases or decreases as a result of regulation.

6 Discussion: Endogenizing returns

In the analysis above, we have kept the return to storage constant and equal to 1 throughout, independently of how many investors choose to hold storage rather than invest in either bank debt or equity. Similarly, we have assumed that the returns to banks' investments are drawn from a fixed distribution that does not depend on how many projects are actually run. Here, we relax both of these assumptions and discuss the implications of doing so for investor returns.

6.1 Endogenizing the return to storage

Our baseline model assumes that the return to storage is always constant, independently of how many households avail themselves of that option. This may be reasonable if one views households' endowments as consisting of nonperishable goods which can be stored "under the mattress", but in other instances supply and demand considerations for assets that deliver consumption in the future may drive their returns. Here, we extend the model to consider the case of where the return to storage is endogenous.

Recall that $S = M - N$ represents households' demand for storage, which represents any asset that delivers 1 unit of consumption at the end of the period. We denote the price of a unit of storage as $P(S)$, where P is an increasing function of S. Denoting the storage asset's yield by δ, so that the gross return is $1 + \delta$, the return to the storage asset can be calculated as $\delta(S) = \frac{1}{P(S)} - 1 = \frac{1-P(S)}{P(S)}$. Clearly, δ will be negative if $P(S) > 1$, which would correspond to the case where there is excess demand for storage assets. Also, δ is increasing in the demand S for storage.

Denote the market equilibrium by the set $\{k^*, N^*, \rho^*, u^*\}$, and consider the effect of imposing a binding capital requirement, $k^{reg} > k^*$, in the context of the frictionless benchmark presented in Section 2. There are two relevant cases: $N^* < M$ and $N^* = M$. Start with the former. In this case, with $k^{reg} > k^*$, the number of banks, N^{reg}, goes down relative to the market equilibrium N^*. This implies that the demand for storage, S, will increase, raising the price $P(S)$ and reducing the yield $\delta(S)$. In other words, the return to storage will go down, and investors that are either users of storage or are depositors will earn a lower return as a result.

Next, consider the case where $N^* = M$. If, under $k^{reg} > k^*$, $N^{reg} < N^*$, then the argument from above applies again and the equilibrium return to storage and deposits goes down. If instead $N^{reg} = N^* = M$, there is then no effect on the demand for storage simply because no storage is being used. For this case, all our results with an exogenous return to storage presented in Proposition 3 continue to hold as stated.

Put together, these results highlight that, when the return to storage is endogenized, the equilibrium return to depositors in the partial inclusion region may decrease as a result of the imposition of binding capital regulation since this pushes more investors into using storage, bidding up its price and as a result lowering its return. Of course, the return to equityholders also decreases, so that the net effect depends on how elastic is the price of the storage asset, $P(S)$.[8]

[8] How much the price of the storage asset reacts to changes in demand, S, may depend on public policies such as monetary policy. For example, if the price of storage is largely determined by the availability of "safe" assets such as Treasury bonds, for instance, the central bank may react to increased demand for storage assets through open market operations by increasing the supply of government bonds. This would have the effect of reducing the slope of the price function P, and compressing the yield changes in the return to storage that accompany changes in demand. Conversely, holding fixed the supply of Treasury bonds would likely maximize the impact of changes in the demand for storage on its yield.

6.2 Endogenizing project returns

We have assumed so far that not only are all banks identical, but also that adding or subtracting banks does not change the expected returns from the projects in which they invest so that there are constant returns to scale at the industry level. But one may well expect the return on investment projects to be a function of the aggregate amount of investment (i.e., a function of the number of banks that operate), declining as more investments are undertaken since there will be more competition for resources, more competitive bidding for projects, or simply lower quality projects on average as the number of projects increases. In this section, we endogenize project returns and, specifically, we allow them to be a decreasing function of the number of banks that operate.

To study this issue, we assume that $E[r] = R$ is a decreasing function of N, so that $\frac{dE[r]}{dN} = R'(N) < 0$. This assumption captures the notion that as more banks enter and, hence, more projects are undertaken, the expected return on each project decreases. For comparability with the previous sections, we maintain the symmetry assumption across banks, so that our assumption amounts to decreasing returns to scale at the industry level. All other parts of the model remain the same.

We first note that when project returns are endogenous and decreasing in N, there is scope for capital regulation to increase social welfare. Indeed, a similar result to that in Proposition 4 obtains, with k^{reg} being optimally greater than k^* when $N^{reg} < N^* < M$. As before, the required increase in capital for each bank will lead to fewer banks entering. However, unlike the cases we have studied so far, the reduction in the number of banks and projects implies that the expected returns of each project will increase given that $\frac{dE[r]}{dN} < 0$. This means that each bank has more surplus to distribute to investors relative to our baseline case where there are constant returns to industry scale (CRS) and $\frac{dE[r]}{dN} = 0$. Since $N^{reg} < M$, we must still have that $u^{reg} = u^* = 1$, but shareholder expected returns, ρ^{reg}, will be larger than in the CRS case. These results imply that the reduction in the number of banks arising from regulation, ΔN, is smaller when project returns are endogenous and decreasing in the number of projects being run, than in the CRS case.

The analysis of the case where $N^* = M$ so that the market is in full inclusion is similar. If regulation is not so strict as to reduce the number of banks, i.e., if $N^{reg} = M$, then the results are identical to those studied in prior sections. If, instead, binding capital regulation pushes $N^{reg} < N^* = M$, the results are summarized in the discussion above, where returns for each project increase and the drop in the number of productive projects that are operated is tempered by the increased return per project, without qualitatively affecting our main results.

7 Capital requirements and recapitalizations

So far we have considered capital requirements that are in place upon inception, so that our analyses represent comparisons of equilibria. From this perspective, our results can be viewed as comparisons across steady states between a banking market that is unregulated and one that is subject to capital regulation. However, the debate concerning increases in capital standards relates to not only long term changes in banks' risk profiles, in terms of what should be the new status quo, but also to how stakeholders of existing banks are likely to be affected by changing capital requirements in the short term. In other words, increases in capital requirements represent, practically speaking, recapitalization exercises on existing banks which have both long and short term consequences.

While our framework is not explicitly about the effect of recapitalizations, it can help shed light on this issue. To see how, consider again our baseline model as in Section 2 and the equilibrium allocation as in Proposition 1 and assume for the moment that banks can only increase capital via an equity issue. For that case, we may view banks as having an initial capital structure $k_0 = k^*$, which may need to be adjusted if a regulator imposes a capital requirement $k^{reg} > k_0$ in an interim period, such as at time $t = \frac{1}{2}$. To the extent that the capital requirement is anticipated by financial institutions, banks in our model would react by complying with the requirement from inception at $t = 0$, rather than delaying to implement the requirement at the interim period. The reason is that there is no value in delaying in the model. Anticipating future recapitalization, investors willing to hold equity would initially require a higher return than in the case of no recapitalization. This would avoid potential conflicts between existing and new shareholders in the future, as well as between shareholders and depositors. In other words, the analysis of an anticipated capital requirement to be applied at $t = \frac{1}{2}$ would mirror our steady state analysis above.

By contrast, an unanticipated recapitalization raises additional issues. Now, banks would be forced to raise capital at $t = \frac{1}{2}$, requiring them to convert some existing depositors into equity holders. This process raises an interesting dynamic that we have not studied in the analysis above. Specifically, existing stakeholders would be required to relinquish part of their claims in order to be able to convince reluctant investors – either current depositors or investors who are not participating in financial markets at all, even through deposits – to hold equity. Importantly, these would be investors who, under the initial pricing of claims, which we denote ρ_0 and u_0, did not find it optimal to hold equity. In order to participate, therefore, these investors would require an expected return $\rho_1 > \rho_0$, setting up a conflict between existing shareholders and new shareholders. If deposit contracts can be reset at $t = \frac{1}{2}$, reflecting the demandable and short term nature of much bank debt, then recontracting will lead deposit rates to come down, reducing the burden that must be borne by the initial shareholders. If, by contrast, the promised deposit payments cannot be renegotiated, as would be the case for term contracts, then any recapitalization would lead to a debt overhang problem. In this case, much of the benefit from the reduction in bankruptcy risk will accrue to the banks' depositors and will come largely at the expense of existing shareholders. As a consequence, existing shareholders would likely oppose leverage reductions, in the spirit of the "leverage ratchet effect" analyzed in Admati et al. (2017).

The discussion above has considered that banks adjust their capital following a regulatory action only via a new issue of equity. Clearly, in practice, banks can also do it through retained earnings. For example, banks that are forced through regulatory intervention to recapitalize may be restricted from paying out dividends to shareholders until capital reaches a sufficiently high level. In our static model, all claims are agreed upon ex ante, and ex post cash flows must be allocated to either one party or the other, with nothing remaining "in the bank." Thus, in a single period model, there is no real role either for retained earnings or dividend restrictions since the latter would be tantamount to a tax paid from profits and given to depositors. To study how retained earnings affect capital structure appropriately therefore calls for a model that incorporates dynamic considerations. We leave this issue for future study.

8 Conclusion

This paper presents an analysis of bank optimal capital structure in a setting where investors, who are heterogenous with respect to their initial endowments, are reluctant to participate in financial markets and have to be induced to do so through the promise of higher returns. The equilibrium amount of market participation in the banking sector is thus endogenous, and depends on the distribution of returns associated with the investment opportunity set available to banks. We use this framework to study the incidence of capital regulation, and shed light on whether capital requirements geared toward reducing bank failure and absorbing losses affect various classes of investors differently.

In conducting our analysis we have abstracted from some important issues, which we believe are important for future research. First, our focus has been on the impact of capital regulation on the sources of financing for the bank, studying which types of investors primarily bear the brunt, or reap the benefits, of regulation. Of course, there are other parties that interact with the bank which are also likely affected by regulation. A salient example is bank borrowers, particularly those that are most financially dependent on their main bank as they may bear part of the cost (or benefit) of regulation through changes in interest rate margins, or through the availability of credit. Likewise, some of the costs and benefits may fall on bank employees.

Second, we have limited the analysis to the case where investors' only alternative to storage is to hold deposits or invest in the financial sector through banks. In practice, of course, there are other institutions, including non-financial firms, with needs for funding and who may wish to raise debt or issue equity. Studying how capital regulation for banks affects the equilibrium distribution of investment and the returns to various financial instruments when such firms are included is certainly an interesting extension.

Finally, in our analysis, we have explicitly sidestepped issues related to the interaction of risk and leverage that are present when systematic risk is priced by assuming risk neutrality. Therefore, the standard results stemming from the work by Modigliani and Miller (1958) are not present, allowing us to isolate the effects stemming from limited market participation and capital regulation. An interesting issue, however, would be to consider how risk aversion, coupled with the existence of systematic risk, interact with the results we obtain here.

A Proofs

Proof of Proposition 1 and Corollary 1.1: We begin by characterizing a bank's capital structure decision, taking ρ and u as given. Consider depositors' participation constraint as given by

$$\int_{r_D(1-k)}^{RX} r_D dG_R(r) = r_D \left(1 - G_R \left(r_D \left(1-k\right)\right)\right) \geq u.$$

In equilibrium, the constraint must bind with equality, and can be solved for k to yield

$$k = 1 - \frac{G_R^{-1}\left(1 - \frac{u}{r_D}\right)}{r_D} = 1 - \frac{B}{r_D},$$

where we have defined $B \equiv G_R^{-1}\left(1 - \frac{u}{r_D}\right)$. Here, B is the bankruptcy threshold, and it can be seen that its definition implies $B = r_D(1-k)$, and that $G_R(B)$ represents banks' probability of bankruptcy. For what follows, it is useful to express k in terms of B only, as

$$k = 1 - \frac{B}{r_D} = 1 - \frac{B(1 - G_R(B))}{u} \tag{17}$$

since $u = r_D \left(1 - G_R \left(B\right)\right)$. Consider now bank excess returns as given in (2). Using (3) and taking the derivative with respect to k yields

$$\frac{dE[\Pi_B]}{dk} = -r_D(1-k)G'(r_D(1-k))\left((1-k)\frac{dr_D}{dk} - r_D\right) - (\rho - u). \tag{18}$$

We can use the implicit function theorem to compute $\frac{dr_D}{dk}$ from (3) as

$$\begin{aligned} \frac{dr_D}{dk} &= -\frac{\frac{\partial}{\partial k}\left(r_D(1 - G_R(r_D(1-k)) - u\right)}{\frac{\partial}{\partial r_D}\left(r_D(1 - G_R(r_D(1-k)) - u\right)} \\ &= -\frac{r_D^2 G_R'(r_D(1-k))}{1 - G_R(r_D(1-k)) - r_D(1-k)G_R'(r_D(1-k))}. \end{aligned}$$

Substituting into (18), and using $B \equiv r_D(1-k)$, we obtain

$$\begin{aligned} \frac{dE[\Pi_B]}{dk} &= u\frac{BG'_R(B)}{1-G_R(B)}\left[\frac{BG'_R(B)+1-G_R(B)-BG'_R(B)}{1-G_R(B)-BG'_R(B)}\right]-(\rho-u) \\ &= \frac{BG'_R(B)}{1-G_R(B)-BG'_R(B)}u-(\rho-u). \end{aligned} \tag{19}$$

Let us define $\overline{B}$ as the unique value of B that satisfies $BG'_R(B) = 1-G_R(B)$. This value is indeed unique because $G_R(\cdot)$ satisfies the increasing hazard ratio property, and hence the term $B\frac{G'_R(B)}{1-G_R(B)}$, which is the product of two monotonically increasing functions, is strictly increasing in B. We can now define $\underline{k} \equiv 1-\frac{\overline{B}(1-G_R(\overline{B}))}{u}$. Note that, for all $B > \overline{B}$, we have $BG'_R(B) > 1-G_R(B)$, whereas for all $B < \overline{B}$ the opposite inequality is true.

Inspection of the first term of (19) reveals that $\frac{dE[\Pi_B]}{dk}$ is strictly negative for $B > \overline{B}$, so no interior equilibrium can ever lie in this region and any equilibrium must have a bankruptcy threshold $B \leq \overline{B}$. We will therefore restrict our analysis to $B \leq \overline{B}$. In this region, B and k move in strictly opposite directions, given that $\frac{dk}{dB} = \frac{BG'_R(B)-(1-G_R(B))}{u} < 0$.

Next, we establish concavity of excess returns in k for all $k > \underline{k}$. Dividing the numerator and denominator of (19) by $1-G_R(B)$, we obtain

$$\frac{dE[\Pi_B]}{dk} = \frac{\frac{BG'_R(B)}{1-G_R(B)}}{1-\frac{BG'_R}{1-G_R(B)}}u-(\rho-u).$$

For $B < \overline{B}$ the first term has an increasing numerator and a decreasing denominator in B. Hence, $\frac{dE[\Pi_B]}{dk}$ is strictly increasing in B for $B < \overline{B}$. Since B and k move in opposite directions, $\frac{dE[\Pi_B]}{dk}$ is decreasing in k for $k \in (\underline{k}, 1]$ and, hence, concave over this range.

We next characterize banks' optimal bankruptcy threshold B^*. Setting (19) to zero and rearranging terms yields the first-order condition

$$\frac{B^*G'_R(B^*)}{1-G_R(B^*)} = \frac{\rho-u}{\rho}, \tag{20}$$

which, by the relation $k^* = 1-\frac{B^*}{r_D} = 1-\frac{B^*(1-G_R(B^*))}{u}$, also uniquely pins down the optimal

k^*, conditional on equilibrium returns ρ and u.

The first-order condition (20) tells us that banks will optimally respond to a higher *relative return gap* $\frac{\rho-u}{\rho}$ by choosing a higher bankruptcy threshold B^*. Moreover, as long as $\rho-u>0$, we see from the first-order condition that $B^*<\overline{B}$. Therefore, the inverse relation between B and k applies always around B^*, implying that optimal capital k^* is a decreasing function of $\frac{\rho-u}{\rho}$.

Investor strategies in partial equilibrium are obtained from (1), which shows that the larger the returns gap $\rho-u$, the more investors will decide to hold equity.

Having established optimal bank and investor decisions for given ρ and u, we proceed to the general equilibrium problem that pins down these returns. For this, we start by assuming that $u>1$. In this case, no investor will wish to invest in storage, and all funds will be invested in banks, with $N=M$.

In equilibrium, banks earn a zero excess return. This implies that, for given ρ, there exists a $u^*(\rho)$ such that $E[\Pi_B]=0$. At $u^*(\rho)$, we can differentiate with respect to ρ to obtain

$$0=\frac{dE[\Pi_B]}{d\rho}=\frac{\partial E[\Pi_B]}{\partial\rho}+\frac{\partial E[\Pi_B]}{\partial u}\frac{du^*}{d\rho}+\frac{\partial E[\Pi_B]}{\partial k}\frac{dk}{d\rho}. \tag{21}$$

The first term is negative, whereas the third term is zero by the Envelope Theorem, given the optimality of k. The second term must therefore be positive for (21) to hold. It comprises two factors: the first factor is $\frac{\partial E[\Pi_B]}{\partial u}=-(1-k)$, which is negative. The second factor must therefore be negative as well: $\frac{du^*}{d\rho}<0$, so that $u^*(\rho)$ is a decreasing function of ρ.

We can now characterize the supply schedule for bank equity, keeping constant the number N of banks. First, observe that both the *return gap* $\rho-u^*(\rho)$ and the *relative return gap* $\frac{\rho-u^*(\rho)}{\rho}$ are monotonic and strictly increasing transformations of ρ. Therefore, since k^* is decreasing in $\frac{\rho-u^*(\rho)}{\rho}$, it is also decreasing in $\rho-u^*(\rho)$. Holding constant the number of banks, we can conclude that aggregate equity supply $S_E=Nk^*$ must be a strictly decreasing function of the return gap $\rho-u^*(\rho)$.

Existence and uniqueness of equilibrium now follows from the fact that equity demand

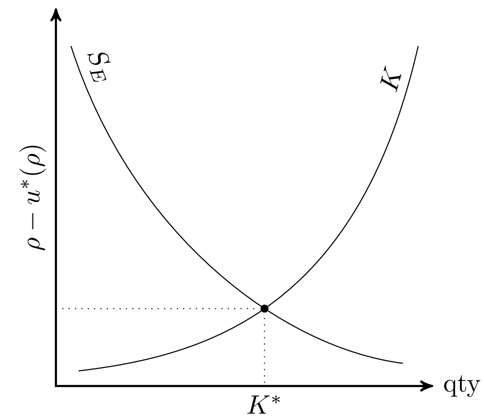

Figure 1: Walrasian Equilibrium and Equilibrium Returns

is upward sloping and equity supply is downward sloping, as functions of the return gap $\rho - u^*(\rho)$ (see Figure 1). Formally, consider the excess demand function for equity, $K - S_E$, which is monotonically increasing in $\rho - u$, clearly negative for $\rho - u \in (0, c/\overline{w})$ (where $K = 0$ and $S_E > 0$), and clearly positive for $\rho - u \geq c/\underline{w}$ (where $K = M$ and $S_E < M$). The intermediate value theorem guarantees the existence of a value $\rho^* - u^*(\rho^*)$ for which excess demand equals zero, and this value is unique because of the monotonicity of the excess demand function. By construction, ρ^* and $u^*(\rho^*)$ represent the model's equilibrium returns under full inclusion.

We next study the comparative statics of the full inclusion equilibrium. Assume that R increases from some initial value R_0 to $R_1 \equiv \lambda R_0$, with $\lambda > 1$. The equity demand schedule, represented by the curve K in Figure 1, is independent of R. Only the equity supply schedule, represented by curve S_E in Figure 1, moves as R is increased from R_0 to R_1.

Consider now how S_E moves. Pick any point $(K_0, \rho_0 - u^*(\rho_0))$ on the original equity supply schedule for $R = R_0$. At this point it is optimal for banks to pick $k_0 = \frac{K_0}{M}$ (and offer some promised return $r_{D,0}$) if $\rho = \rho_0$ and $u = u_0 = u^*(\rho_0)$. Moreover, at this allocation, banks' excess return must equal zero, given the definition of u^*. In other words, every point on the original supply schedule still satisfies all conditions for equilibrium *except for* market clearing. We now show that for every such point on the supply curve with $R = R_0$, there is a one-to-one mapping to a corresponding point on the supply curve for $R = R_1$, and that this

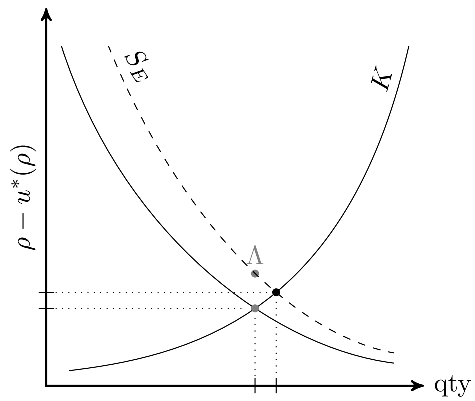

Figure 2: Upwards shifted equity supply curve following an increase in R

new point continues to satisfy the relevant optimality and zero excess return conditions.

In particular, consider a strategy under $R = R_1$ in which banks choose capital $k_1 = k_0$ and promise depositors $r_{D,1} = \lambda r_{D,0}$ whenever $u_1 = \lambda u_0$ and $\rho_1 = \lambda \rho_0$. With this strategy, each bank's bankruptcy threshold $B_1 = r_{D,1}(1 - k_1) = \lambda r_{D,0}(1 - k_0) = \lambda B_0$ increases by the exact amount that is needed to keep the probability of bankruptcy the same as before, under $R = R_0$: $G_{R_1}(B_1) = F(B_1/R_1) = F(\lambda B_0/(\lambda R_0)) = G_{R_0}(B_0)$. On the other hand, the density $G'_{R_1}(B_1) = \frac{1}{R_1} F'(B_1/R_1) = \frac{1}{\lambda R_0} F'(\lambda B_0/(\lambda R_0)) = G'_{R_0}(B_0)/\lambda$ decreases by a factor $\frac{1}{\lambda}$. Since this exactly offsets the increase in the bankruptcy threshold B, the left-hand side of (20), $\frac{BG'(B)}{1-G(B)}$, remains constant after the change in R. The right-hand side of (20), $\frac{\rho - u}{\rho}$, remains constant as well since the common factor λ in returns cancels out. Hence, (20), continues to be satisfied for this new allocation under $R = R_1$. Moreover, the depositor participation constraint remains satisfied as well, $r_{D,1}(1 - G_{R_1}(B_1)) = \lambda r_{D,0}(1 - G_{R_0}(B_0)) = \lambda u_0 = u_1$, and it is easy to verify that excess returns under this new allocation are proportionally greater exactly by a factor of λ and thus continue to equal zero. In summary, for every point $(K_0, \rho_0 - u_0^*(\rho_0))$ on the equity supply curve for $R = R_0$ there exists a corresponding point $(K_0, \lambda(\rho_0 - u_0^*(\rho_0)))$ on the equity supply curve for $R = R_1$, with $\lambda > 1$. The new equity supply curve is illustrated in Figure 2.

We can now establish the comparative statics of k in full inclusion. As illustrated by point Λ in Figure 2, if banks were to keep their capital structure constant as R increases, the shifted equity supply curve would lead to a greater return gap, which would create excess demand for equity. Clearly, this can't be an equilibrium. To balance equity supply with increased demand, aggregate equity issuance K^* must increase as R increases, and the return gap $\rho - u^*(\rho)$ must increase with R as well. Furthermore, under full inclusion the equity of an individual bank $k^* = \frac{K^*}{M}$ is just a constant fraction of aggregate equity, and it is therefore also increasing in R.

Next, we argue that under full inclusion the relative return gap $\frac{\rho - u}{\rho}$ is decreasing in R. To see why, consider just as before an increase from R_0 to $R_1 = \lambda R_0$. For $R = R_0$ at equilibrium the relative return gap is $\frac{\rho_0 - u_0}{\rho_0}$. This value would remain exactly the same if instead $R = R_1$ but banks chose the capital structure that was optimal for $R = R_0$ (represented as the off-equilibrium point Λ in Figure 2) since $\frac{\rho_1 - u_1}{\rho_1} = \frac{\lambda(\rho_0 - u_0)}{\lambda \rho_0} = \frac{\rho_0 - u_0}{\rho_0}$. However, the actual equilibrium for $R = R_1$ features higher capital than this off-equilibrium point. Moving along the $R = R_1$ equity supply curve to the actual equilibrium point, the first order condition (20) must continue to hold. This argument establishes that a lower bankruptcy threshold B and thus a lower value for $\frac{BG'(B)}{1-G(B)}$ implies also a lower relative return gap $\frac{\rho_1 - u_1}{\rho_1}$ in equilibrium.

Next, we establish the comparative statics results. Since in equilibrium $E[\Pi_B] \equiv 0$ for all R, it must be that

$$\frac{d}{dR}\left[\int_{r_D(1-k)}^{RX} r dG_R(r) - u(1-k) - \rho k\right] = 0$$

when k maximizes bank profits. Consider, however, that $\frac{dE[\Pi_B]}{dR} = \frac{\partial E[\Pi_B]}{\partial R} + \frac{\partial E[\Pi_B]}{\partial k}\frac{dk}{dR} = \frac{\partial E[\Pi_B]}{\partial R}$ since $\frac{\partial E[\Pi_B]}{\partial k} = 0$ by the Envelope Theorem. This implies that $\frac{\partial}{\partial R}\left[\int_{r_D(1-k)}^{RX} r dG_R(r)\right] - (1 - k)\frac{du}{dR} - k\frac{d\rho}{dR} = 0$ or, equivalently,

$$\frac{\partial}{\partial R}\left[\int_{r_D(1-k)}^{RX} r dG_R(r)\right] = (1-k)\frac{du}{dR} + k\frac{d\rho}{dR}. \tag{22}$$

The expression on the left-hand side of this equation is positive. To see why, first note that, keeping the default boundary, $r_D(1-k)$, constant, $G_{R'}(r) < G_R(r)$ for all r for $R' > R$. Moreover, the deposit rate, r_D, is affected by changes in R only through changes in u. If u

were to be decreasing in R, this would lower r_D and make the term on the left hand side larger. This implies that $\frac{\partial}{\partial R}\left[\int_{r_D(1-k)}^{RX} r dG_R(r)\right]$ could only be negative if u were to increase sufficiently when R increases. But note that from earlier results we know that the return gap $\rho - u$ must increase as R increases, so if u increases in R then ρ must increase by even more. Since the right hand side, $(1-k)\frac{du}{dR}+k\frac{d\rho}{dR}$, is simply how the change in the total project return gets allocated, the increase in both ρ and u implies a contradiction to $\frac{\partial}{\partial R}\left[\int_{r_D(1-k)}^{RX} r dG_R(r)\right]$ being negative. It must therefore be that $\frac{\partial}{\partial R}\left[\int_{r_D(1-k)}^{RX} r dG_R(r)\right] > 0$.

Given the left hand side of (22) is positive, the right-hand side must be positive as well, which is impossible if both $\frac{du}{dR}$ and $\frac{d\rho}{dR}$ are negative. So either u or ρ (or both) must be strictly increasing in R. An increase in u and fall in ρ would contradict the fact that $\rho - u$ increases with R. Similarly, an increase in ρ and a decrease in u contradicts the finding that the relative return gap $\frac{\rho-u}{\rho}$ is a decreasing function of R. Hence, the only remaining possibility must hold, i.e., both ρ and u must be increasing in R.

Having shown that u increases in R in the full inclusion region, it becomes clear that for R sufficiently small any full inclusion equilibrium must violate the condition $u \geq 1$. To see this, consider the limit $R \to 1^+$ such that $E[r] \to 1^+$. Under full inclusion, banks always choose an interior amount $k^* > 0$ of equity, and thus, due to market clearing, in equilibrium $\rho^* > u^*$. Suppose $u^* \geq 1$. Then for any $k > 0$ excess returns would be negative since in the limit $E[r] \to 1$, yielding a contradiction. Hence, for sufficiently small R, full inclusion would imply that $u^* < 1$, violating depositors' participation constraint. Thus, a full inclusion equilibrium where $N = M$ cannot exist for R low enough.

We define $\overline{R}$ as the unique value of R for which $u^* = 1$ in full inclusion. Due to the monotonicity of u^* in R, a full inclusion equilibrium exists for all $R \geq \overline{R}$. For all $R < \overline{R}$, only a partial inclusion equilibrium with $N < M$ exists. For this case, since $N < M$, investors must be indifferent between storage and deposits, which means that $u = 1$. Following virtually the same argument as for the full inclusion case, ρ can be shown to be increasing in R in this region. This in turn implies that the right-hand side of (20), $\frac{\rho-u}{\rho}$, must be *increasing* in R

under partial inclusion. Therefore, by the first order condition, the bankruptcy threshold B^* and the bankruptcy probability $G_R(B)$ are all increasing in R or, equivalently, k is decreasing in R.

Market clearing now determines the number of active banks: since $\rho^* - u$ is an increasing function of R, the aggregate amount of capital K increases in R whereas capital at the bank level, k, is decreasing. It follows that there exists a market clearing number of active banks N which increases with R.

As the last step, we establish that, in all cases, $\rho^* > E[r] > u^*$. First, it is straightforward to see that $k^* > 0 \Leftrightarrow \rho^* > u^*$, as otherwise the market for equity could never clear. One therefore need only check that $k = 0$ cannot be an optimum. To see why $k = 0$ cannot be optimal, consider a bank that chooses $k = 0$ but $r_D < RX$. In that case, ρ, the expected return per unit of capital, will be unboundedly large, contradicting that $k = 0$ clears the market. Suppose instead that $r_D = RX$. In that case, with $k = 0$, the bank would go bankrupt with probability 1, making it impossible to satisfy depositors' participation constraint. Therefore, k^* must be strictly positive in any equilibrium, implying that $\rho^* > u^*$.

For the next part, clearly $\rho^* > u^* \geq E[r]$ is not possible as otherwise the bank would be paying out to investors more than what it is able to produce from its projects. Note also that $\rho^* < E[r]$ cannot be consistent with equilibrium since, if it were, any bank could instead decide to be all equity financed (i.e., set $k = 1$), have no probability of bankruptcy, and deliver an expected return of $E[r]$ to its shareholders, contradicting that assumption that $\rho^* < E[r]$. Therefore, it must be that $\rho^* > E[r] > u^*$ holds in any equilibrium. □

For all subsequent proofs, we suppress the superscripts $*$ for equilibrium variables, except where there is risk of confusion.

Proof of Corollary 1.2: Since equity supply is derived from the bank's maximization problem, it is unaffected by a change in c. By contrast, aggregate equity demand comes from investors' maximization problem, and therefore it declines as c increases (eq. 1). Consider now the relation between the equilibrium returns for shareholders and depositors, obtained from the zero profit condition for banks, (4), and characterized as $u^*(\rho)$ in the proof of Proposition 1. For the purposes of this result, it is useful to abuse notation slightly and express this equivalently as $\rho^*(u)$, meaning the value of ρ that yields the bank zero profits for any given value of u. Since ρ^* is obtained entirely from the bank's zero profit constraint, it is not a function of c, only of u. Since when $N < M$, $u = 1$, it follows that ρ^* will be independent of c within this region. However, since aggregate demand for equity, given by $\int_{\frac{c}{\rho-u}}^{\overline{w}} wdH(w)$, is now lower, the market for equity can only clear if N decreases. This establishes the first part of the result.

To establish the second part, note that for $N = M$ we have $u > 1$. The leftward shift of the equity demand curve yields a new equilibrium point $\rho^* - u^*(\rho^*)$ that is characterized by a higher return gap and lower aggregate capital. Note that banks' excess returns do not depend on c directly, but only indirectly via the response of the endogenous variables ρ and u to changes in c. Hence, the zero excess return condition imposes that u and ρ must move in opposite ways, $\frac{du^*}{d\rho} < 0$. Hence, ρ^* must be increasing in c whereas u^* is decreasing.

Finally, to see that $\overline{R}$ is increasing in c, note that, for $E[r] = \overline{R}(c)$, where we highlight the dependence of the threshold $\overline{R}$ on the cost c, u is still equal to 1 since we are at the threshold, so that an increase in c to c' will lead to a reduction in N from $N = M$ to $N' < M$. This establishes that the threshold for full inclusion must increase, namely that $\overline{R}(c') > \overline{R}(c)$ for $c' > c$. □

Proof of Proposition 2: To establish the result, we start with the problem of bank excess return maximization as in (2), subject to the same constraints as above for the social planner's problem. Given that depositors' participation constraint in (3) will always be satisfied with equality, we can substitute it into the bank's maximization problem to obtain

$$\max_k E\left[\Pi_B\right] = \int_{r_D(1-k)}^{RX} rdG_R\left(r\right) - u(1-k) - \rho k.$$

The necessary first order condition that must now be satisfied is

$$g_R\left(r_D(1-k)\right)\left(r_D^2\left(1-k\right)-\frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right)-\left(\rho-u\right)=0,$$

where $\frac{\partial r_D}{\partial k}$ is obtained from depositors' participation constraint, and $g_R=G_R'$.

Consider now the maximization problem for the social planner, which can be written as

$$\max_k SW = N\int_{r_D(1-k)}^{RX} r dG_R\left(r\right)+M-N-C,$$

where $C=\int_{\widehat{w}}^{\overline{w}} cdH\left(w\right)$ is the total costs of participation that are incurred. The necessary first order condition for this problem is

$$N\left(r_D^2\left(1-k\right)g_R\left(r_D(1-k)\right)-\frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D g_R\left(r_D(1-k)\right)\right)+\frac{\partial N}{\partial k}\int_{r_D(1-k)}^{RX} r dG_R\left(r\right)-\frac{\partial N}{\partial k}-\frac{\partial C}{\partial k}=0.$$

The main issue is to characterize $\frac{\partial C}{\partial k}$. For this, it is useful to integrate C to obtain $C=\int_{\widehat{w}}^{\overline{w}} cdH\left(w\right)=c\left(H\left(\overline{w}\right)-H\left(\widehat{w}\right)\right)=c\left(1-H\left(\widehat{w}\right)\right)$ since $H\left(\overline{w}\right)=1$. Now, recall that the investor demand for equity is given by $K=\int_{\widehat{w}}^{\overline{w}} wdH\left(w\right)$. Integrate by parts to express K as:

$$\begin{aligned} K &= \int_{\widehat{w}}^{\overline{w}} wdH\left(w\right)=wH\left(w\right)|_{\widehat{w}}^{\overline{w}}-\int_{\widehat{w}}^{\overline{w}} H\left(w\right)dw \\ &= \overline{w}-\widehat{w}H\left(\widehat{w}\right)-\int_{\widehat{w}}^{\overline{w}} H\left(w\right)dw \end{aligned}$$

since $H\left(\overline{w}\right)=1$. In equilibrium, the market clearing condition implies that $K=kN$. Equating the above to kN and rearranging gives us

$$\widehat{w}H\left(\widehat{w}\right)=\overline{w}-\int_{\widehat{w}}^{\overline{w}} H\left(w\right)dw-kN.$$

Now, since $\widehat{w} = \frac{c}{\rho - u}$, we can rewrite yet again as

$$\frac{c}{\rho - u} H(\widehat{w}) = \overline{w} - \int_{\widehat{w}}^{\overline{w}} H(w)\, dw - kN \Rightarrow cH(\widehat{w}) = (\rho - u)\left(\overline{w} - \int_{\widehat{w}}^{\overline{w}} H(w)\, dw - kN\right).$$

Since $C = c(1 - H(\widehat{w}))$, we can use this to rewrite C as

$$C = c - (\rho - u)\left(\overline{w} - \int_{\widehat{w}}^{\overline{w}} H(w)\, dw - kN\right).$$

With this, we establish that

$$\frac{\partial C}{\partial k} = (\rho - u) N + (\rho - u) k \frac{\partial N}{\partial k}.$$

We can now substitute $\frac{\partial C}{\partial k}$ into the FOC for the social planner and obtain

$$\begin{aligned} & N\left(r_D^2 (1-k)\, g_R(r_D(1-k)) - \frac{\partial r_D}{\partial k}(1-k)^2 r_D g_R(r_D(1-k))\right) \\ & + \frac{\partial N}{\partial k} \int_{r_D(1-k)}^{RX} r dG_R(r) - \frac{\partial N}{\partial k} - (\rho - u) N - (\rho - u) k \frac{\partial N}{\partial k} = 0. \end{aligned}$$

Grouping terms yields

$$\begin{aligned} & N\left(g_R(r_D(1-k))\left(r_D^2(1-k) - \frac{\partial r_D}{\partial k}(1-k)^2 r_D\right) - (\rho - u)\right) \\ & + \frac{\partial N}{\partial k}\left(\int_{r_D(1-k)}^{RX} r dG_R(r) - 1 - (\rho - u) k\right) = 0. \end{aligned}$$

Now observe that $\frac{\partial N}{\partial k} \neq 0 \Rightarrow u = 1$ since, if $u > 1$, all funds are being used in the banking sector, so a marginal increase in k cannot change $N = M$.

Consider first the case that $u > 1$, so that $\frac{\partial N}{\partial k} = 0$. We are then left with only

$$N\left(g_R(r_D(1-k))\left(r_D^2(1-k) - \frac{\partial r_D}{\partial k}(1-k)^2 r_D\right) - (\rho - u)\right) = 0,$$

which is the same condition as must be satisfied for the bank's problem.

Alternatively, suppose that $u = 1$, which allows us to express the first order condition for the social planner as

$$N\left(g_R\left(r_D(1-k)\right)\left(r_D^2\left(1-k\right)-\frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right)-\left(\rho-u\right)\right)$$
$$+\frac{\partial N}{\partial k}\left(\int_{r_D(1-k)}^{RX} r\, dG_R\left(r\right)-\left(1-k\right)-\rho k\right)=0.$$

Note now that the term in the parentheses of the second line is simply equal to $E\left[\Pi_B\right]$ for the case where $u = 1$, which in equilibrium is equal to zero, with all rents going to shareholders through ρ. This leaves the term below, after eliminating the N:

$$g_R\left(r_D(1-k)\right)\left(r_D^2\left(1-k\right)-\frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right)-\left(\rho-u\right)=0,$$

again exactly as in the bank's problem for the case $u = 1$. Therefore, for all cases the necessary condition to be satisfied is identical to that which maximizes excess returns. Given that the market clearing condition is the same across both maximization problems, we conclude that both problems must have the same solution. □

Proof of Proposition 3: Recall the bank's maximization problem, given in (2), (3), (4), and (5). Given $E[\Pi_B] = 0$ in equilibrium, (2) can be rewritten as

$$E\left[r|r>r_D(1-k)\right]-u(1-k)-\rho k=0.$$

We now differentiate this expression with respect to k to obtain

$$\frac{d}{dk}E\left[r|r>r_D(1-k)\right]-\frac{du}{dk}(1-k)+u-\frac{d\rho}{dk}k-\rho=0,$$

or, equivalently,

$$\rho-u=\frac{d}{dk}E\left[r|r>r_D(1-k)\right]-\frac{du}{dk}(1-k)-\frac{d\rho}{dk}k.$$

Evaluated at the equilibrium value of capital, $\frac{d}{dk}E\left[r|r > r_D(1-k)\right] = \rho - u$, which is just the FOC for the bank's problem. For greater amounts of capital, $\frac{d}{dk}E\left[r|r > r_D(1-k)\right] < \rho - u$ since the level of capital exceeds what is privately optimal. Substituting and simplifying yields

$$-\frac{du}{dk}(1-k) - \frac{d\rho}{dk}k = \rho - u - \frac{d}{dk}E\left[r|r > r_D(1-k)\right] > 0,$$

which implies that $\frac{du}{dk}(1-k) + \frac{d\rho}{dk}k < 0$.

Now consider the case where $N^* < M$, so that $u = 1$. Here, $\frac{du}{dk} = 0$, so that we must have $\frac{d\rho}{dk}$ in order to satisfy the free entry condition (i.e., zero profit) condition for the banks. Since ρ decreases as a result of regulation, but $u = 1$, this means that $\rho - u$ goes down and hence the total amount of capital investors are willing to hold, K, must decrease as well. Combined with $k > k^*$, this means that N must decrease: $N^{reg} < N^*$.

For the case where $N^* = M$ and a local increase in capital around the market equilibrium, so that N does not decrease, the differential in returns is pinned down by market clearing: banks' total supply of equity capital is $K = kM$, where $k > k^*$. From the investors' problem, we know that total demand for equity is given by $\int_{\frac{c}{\rho-u}}^{\overline{w}} wdH(w)$. From this, it is immediate that in order to satisfy the capital requirement we need $\rho - u$ to increase relative to the market equilibrium, $\rho^* - u^*$. To see that u must decrease, note from above that as k increases, in equilibrium we must have $\frac{du}{dk}(1-k) + \frac{d\rho}{dk}k < 0$. The fact that there must be an increase in $\rho - u$ to satisfy market clearing rules out the possibility that $\frac{d\rho}{dk} < 0$ but $\frac{du}{dk} > 0$. Therefore, whether ρ increases or decreases in k, we must have that $\frac{du}{dk} < 0$. □

Proof of Proposition 4: The proof follows a similar approach as that of Proposition 2. Since at equilibrium (8) will be satisfied with equality, we can rewrite (2) as

$$\max_k E\left[\Pi_B\right] = \frac{1}{2R}\int_{r_D(1-k)}^{2R} rdr + \frac{1}{2R}\int_0^{r_D(1-k)} hrdr - u(1-k) - \rho k.$$

The necessary first order condition that must now be satisfied is

$$\frac{1}{2R}\left(r_D^2(1-k)-\frac{\partial r_D}{\partial k}(1-k)^2 r_D\right)-(\rho-u)+h\frac{1}{2R}\left(\frac{\partial r_D}{\partial k}(1-k)^2 r_D-r_D^2(1-k)\right)=0,$$

or

$$\frac{1}{2R}\left(r_D^2(1-k)-\frac{\partial r_D}{\partial k}(1-k)^2 r_D\right)(1-h)-(\rho-u)=0,$$

where $\frac{\partial r_D}{\partial k}$ is obtained directly from (8).

Consider now the social planner that choose capital to maximize (9). As above, define $C\equiv\int_{\widehat{w}}^{\overline{w}} cdH(w)$. For an interior solution, the necessary first order condition to this problem is

$$N\frac{1}{2R}\left(r_D^2(1-k)-\frac{\partial r_D}{\partial k}(1-k)^2 r_D\right)+\frac{\partial N}{\partial k}\frac{1}{2R}\int_{r_D(1-k)}^{2R} rdr-\frac{\partial N}{\partial k}$$
$$+Nh\frac{1}{2R}\left(\frac{\partial r_D}{\partial k}(1-k)^2 r_D-r_D^2(1-k)\right)+\frac{\partial N}{\partial k}\frac{1}{2R}\int_0^{r_D(1-k)} hrdr+N\frac{1}{2R}\int_0^{r_D(1-k)}\frac{\partial h}{\partial k}rdr-\frac{\partial C}{\partial k}=0.$$

Using the same arguments as above in the proof of Proposition 2, we can show that

$$\frac{\partial C}{\partial k}=(\rho-u)N+(\rho-u)k\frac{\partial N}{\partial k}.$$

We can now follow the same argument as above to show that, for the case where $u>1$ so that $\frac{\partial N}{\partial k}=0$ In that case, $\frac{\partial h}{\partial k}=0$, and there is thus no scope for capital regulation.

By contrast, for the case where $u=1$ and $\frac{\partial N}{\partial k}\neq 0$, so that $\frac{\partial h}{\partial k}\neq 0$ as well. Indeed, for this case algebraic manipulations that the FOC for the social planner differs from that of the banks by the term $N\frac{1}{2R}\int_0^{r_D(1-k)}\frac{\partial h}{\partial k}rdr$, which is strictly positive. Therefore, the market solution involves banks holding too little capital relative to what a social planner would like and, consequently, too many banks. □

Proof of Corollary 4.1: Since $k^{reg}>k^*$ but $N^{reg}<N^*$, the comparison of K^{reg} relative to K^* is at first glance ambiguous. However, since the increased capital requirement leads to less

leverage at each bank, bankruptcy costs at each bank are lower. In addition, the reduction in the number of banks to N^{reg} further reduces bankruptcy costs through the greater recovery: $h' < 0$. Thus, a decrease in K and ρ would be inconsistent with an increase in SW since all the surplus goes to either depositors or shareholders. For $N^{reg} < M$, leverage $1-k$ goes down at each bank, and depositors earn their outside options so that $u = 1$. Hence, shareholders must be capturing the increase in surplus, implying that ρ and K increase relative to the market solution. □

Proof of Proposition 5: The first order condition for (10) is

$$\frac{1}{2R}\left(2R - r_D\left(1-k\right)\right)\left(r_D - \frac{\partial r_D}{\partial k}\left(1-k\right)\right) - \rho = 0.$$

With $\gamma > 0$, one can see from depositors' participation constraint that the promised interest rate on deposits is lower than when there is no insurance. Moreover, $\frac{\partial r_D}{\partial k} < 0$ and becomes smaller in magnitude (i.e., attenuates toward 0) as γ increases since the deposit rate becomes less sensitive to changes in capital k because of the insurance. Combined, both of these effects shift the first order condition down, for all values of ρ and u. From the results on the comparison of equilibria from Milgrom and Roberts (1994), this implies that the solution to (10) will yield a lower level of capital than k^*, the solution when there is no deposit insurance.

To see that k^γ is decreasing in γ, simply note that further increases in γ shift the FOC down even more, leading to lower equilibrium values of capital k^γ, so that $\frac{\partial k^\gamma}{\partial \gamma} < 0$. □

Proof of Proposition 6: To establish this result, it is useful to substitute depositors' participation condition, (11), into the bank's problem, (10), to obtain

$$\max_k E\left[\Pi_B\right] = \frac{1}{2R}\int_{r_D(1-k)}^{2R} r dr + \gamma r_D\left(1-k\right)\left(\frac{r_D\left(1-k\right)}{2R}\right) - u\left(1-k\right) - \rho k.$$

In other words, banks, and hence shareholders, benefit precisely by the size of the expected deposit insurance payment. A social planner, however, would only consider the total output, as given in (12). The difference is precisely the term $\gamma r_D (1-k) \left(\frac{r_D(1-k)}{2R}\right)$, times the number of banks N, so that the planner's FOC with respect to k differs from that for a bank by the term

$$\frac{\partial}{\partial k}\left(-\gamma r_D (1-k) \left(\frac{r_D (1-k)}{2R}\right) N\right).$$

The derivative of this term is positive since an increase in k reduces the size of the deposit insurance subsidy, as well as the deposit rate. This means that the planner's first order condition is an upward shift of the first order condition for a bank and, again from the results on comparing equilibrium in Milgrom and Roberts (1994), the solution to the planner's problem must entail a higher level of capital than the solution to the bank's problem.

The second part of the result follows from an argument nearly identical to that in the proof of Proposition 3, where we consider the required change in either ρ or u, or both, to maintain zero profits given the requirement that k be greater than k^γ. It can then readily be shown that for $E[r] \leq \overline{R}$, capital regulation reduces ρ and N, while for $E[r] > \overline{R}$, market clearing implies that the gap between shareholder and depositor returns must widen: $\frac{\partial(\rho-u)}{\partial k} > 0$ for $k > k^\gamma$. □

Proof of Corollary 6.1: To establish the result, consider again the case where there is no deposit insurance as in Section 3 and denote the allocation for capital as k^*, with corresponding u^*, ρ^*, and N^*. Suppose now that, when there is deposit insurance, the social planner were to set $k^{reg} = k^* > 0$. This capital requirement will be binding since, as shown in Proposition 5 banks prefer to hold $k^\gamma < k^*$ when deposits are insured. There are two cases now to consider:

1) Suppose that $N^* = M$. In this case, it must be that $\rho^* - u^* = \rho^{reg} - u^{reg}$ since $K^* = Mk^* = Mk^{reg} = K^{reg}$. It follows that either $\rho^* \geq \rho^{reg}$ or $\rho^* < \rho^{reg}$ must hold. Suppose first that $\rho^* \geq \rho^{reg}$ and thus $u^* \geq u^{reg}$. This implies $r_D^* > r_D^{reg}$, since $r_D^* > u^* \geq u^{reg}$, so that bankruptcy costs are strictly lower with deposit insurance. Hence, SW^{reg} must be higher than SW^*, leading to a contradiction in $\rho^* \geq \rho^{reg}$, so that we must have $\rho^* < \rho^{reg}$, and

$u^* < u^{reg}$.

2) Suppose instead that $N^* < M$ so that $u^* = 1$. It follows that $u^{reg} \geq u^*$ must hold. Suppose now that $N^{reg} \leq N^*$. Then $u^{reg} = u^* = 1$, and $r_D^{reg} < r_D^*$. But then expected bankruptcy costs must be lower at each bank when deposits are insured and, since $N^{reg} \leq N^*$, they must be lower in aggregate. This implies that $\rho^{reg} > \rho^* \Rightarrow K^{reg} > K^*$. But since $k^{reg} = k^*$, it would then have to be that $N^{reg} > N^*$, which is a contradiction to $N^{reg} \leq N^*$. Therefore, it must be that $N^{reg} > N^*$, and SW is higher under deposit insurance when $k^{reg} = k^* > 0$.

Finally, since this is true for $k^{reg} = k^*$, it must a fortiori be true that SW will be higher when capital regulation is set optimally. Clearly, choosing $k = 0$ is not optimal, so that optimally we must have $k^{reg} > 0$. □

Proof of Lemma 7: As a preliminary result, note that, if $\gamma = 0$ so that there is no deposit insurance coverage, and if $a = 0$ so that there is no risk shifting, $r_D(1-k) \leq E[r] = R$ in equilibrium. To see why, observe that depositors' payoff would be $\frac{1}{2R}\int_{r_D(1-k)}^{2R} r_D dr = u$. This expression is maximized at $r_D(1-k) = R$, whereas bank profits are strictly decreasing in r_D. Hence, any solution that maximizes bank profits must have $r_D(1-k) \leq R$, for any value of u. Now, deposit insurance will lower r_D further. As well, allowing for risk shifting does not change the conclusion that depositors' payoff is maximized at some $r_D(1-k) \leq R$.

Now consider the bank's FOC with respect to the degree of risk shifting a, given by

$$-\frac{1}{2R}\int_{r_D(1-k)}^{2R} (r - r_D(1-k))\, dr + \frac{1}{2}(2R - r_D(1-k)) - a = 0.$$

We can differentiate this FOC with respect to k to obtain the effect of an increase in capital on the bank's choice of a. Note that

$$\frac{d}{dk}\left(\frac{\partial \Pi_B}{\partial a}\right) = \frac{\partial^2 \Pi_B}{\partial k \partial a} + \frac{\partial^2 \Pi_B}{\partial r_D \partial a}\frac{dr_D}{dk}.$$

Hence,

$$\frac{\partial^2\Pi_B}{\partial k \partial a} + \frac{\partial^2\Pi_B}{\partial r_D \partial a}\frac{dr_D}{dk} = -\frac{1}{4R} r_D \left(2R - 2r_D\left(1-k\right)\right) + \left(1-k\right)\frac{dr_D}{dk}\left(\frac{1}{2R}\int_{r_D(1-k)}^{2R} dr - \frac{1}{2}\right)$$

The first term is negative since $2R > 2r_D\left(1-k\right)$. For the second term, rewrite the integral portion as $\frac{2R - r_D(1-k)}{2R} - \frac{1}{2}$, which is also positive. So the sign of the second term depends on the sign of $\frac{dr_D}{dk}$. To sign that term, focus again on depositors' expected utility, for the case where $\gamma = a = 0$ (the same argument applies if either of these are positive). Observe that

$$\frac{d}{dk}\left(\frac{1}{2R} r_D\left(2R - r_D\left(1-k\right)\right)\right) = \frac{1}{2R} r_D^2,$$

which is strictly positive, meaning that the payout to depositors increases in k, for given r_D. Now consider the derivative with respect to r_D, which is

$$\frac{d}{dr_D}\left(\frac{1}{2R} r_D\left(2R - r_D\left(1-k\right)\right)\right) = \frac{1}{R}\left(R - r_D\left(1-k\right)\right).$$

This is also strictly positive, for all k. Put together, we see that in order to satisfy depositors' participation constraint with equality as k increases, r_D must go down, i.e., $\frac{dr_D}{dk} < 0$. The argument now implies that if the bank had more capital, it would choose a lower a. □

Proof of Proposition 8: The social planner's problem, (15), differs from the bank's problem precisely by the two terms highlighted above, $\gamma\frac{1-a}{2R}\int_0^{r_D(1-k)} r_D\left(1-k\right) dr$ and $\gamma\frac{a}{2} r_D\left(1-k\right)$, each multiplied by the number of banks N. Both of these terms enter negatively into the social planner's problem, and are reduced directly by increases in k, and indirectly through the effect of higher k in reducing r_D. Therefore, it is straightforward to conclude that $k^{reg} > k^*$, and that $SW^{reg} > SW^*$.

The argument that $\rho - u$ must increase under capital regulation relative to the market solution, for $N^{reg} = M$, follows from market clearing in a similar fashion to the proof of Proposition 3. □

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