Geeks Bearing Gifts:
Competition and Information Sharing
in Knowledge-Based Service Industry∗

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Abstract

In this paper, we build a model of a knowledge-based service industry focusing on customer collaboration and its dynamic feedback on the stock of knowledge of a service firm. We apply our service-based approach to explain why firms that capitalize on software-related services may want to release their software as open source. More precisely, we consider two variants of a general model. When the customer makes an ex ante investment that enhances her collaboration, we find that knowledge sharing (through open source, for instance) and/or market sharing can be a strategy that a dominant firm employs to boost the investment. When the project size is exogenous but the customer chooses collaboration level after selecting a service firm, we find that open source may be an aggressive entry strategy and the dominant firm may either voluntarily choose open source to boost collaboration or be forced to embrace open source in order not to lose competition.

Keywords: open source, service industry, dynamics, customer collaboration, customer-specific knowledge

JEL classifications: L17, L84, L86, D86

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1 Introduction

The endorsement of open source software development principles by big software firms such as Sun Microsystems and IBM has changed the face of software industry dramatically during the past ten years. As late as in the late nineties, it was generally understood that the source code of virtually any commercial software project was an intellectual asset which had to be guarded carefully against unauthorized dissipation because it contained core elements of business-relevant knowledge of the firm. This thought was challenged in Eric Raymond’s influential book “The Cathedral and the Bazaar” (Raymond, 1999): impressed by the tremendous success of the volunteer-driven open development models around the Linux operating system, he advocated the general adoption of a more open development model in software industry.

Today, profit-maximizing firms such as Microsoft and IBM are actively developing and distributing intellectual assets of billions of dollars of worth under open source licenses. “Open source” implies that the entire product’s source code is freely available for download under a license that permits unlimited use, copying, modification and redistribution of the code and any derived work, thus making the product a public good. Examples of commercially developed open source products include the Java language, the OpenOffice software suite, and the JBoss application server. The commercial momentum behind open source software nowadays even extends to projects that used to be driven by volunteer contributions: According to a study by Kroah-Hartman et al. (2008), over 70 percent of the more recent contributions to the Linux kernel were made by programmers who were paid for their work.

To the economist, the most immediate question is which incentives lie behind this free provision of a public good. Incentives for participation in open source software development have so far been discussed primarily in the context of individual non-professional contributors, for instance by Lerner and Tirole (2002). The literature currently only offers a handful of explanations why profit-maximizing firms might want to participate in open source software development at as high intensity as empirically documented. Existing arguments include the sale of complementarities (Lerner and Tirole, 2005), positive network externality to a substitute closed source product (Mustonen, 2005), the higher profitability of an open source platform in two-sided markets where applications are proprietary (Economides and Katsamakas, 2006), and the option to generate revenues through dual-licensing (Comino and Manenti, 2007). However, neither of these approaches offers a compelling explanation for the numerous cases in which the commercial open source producer does not control any complementary market to generate revenue, and in which the deployed software is not a platform or a complement to a commercial offer of the firm.

In this paper, we embark on a different path and take Eric Raymond’s provocative assessment that “...software is largely a service industry operating under the persistent but unfounded delusion that it is a manufacturing industry” (Raymond, 1999, p. 120) as an invitation to characterize the general properties of competition in a knowledge-based service industry: We aim to see whether commercial open source software development can be
understood more naturally from the perspective of a competitive service sector.

Indeed, knowledge-based service firms operate very differently from manufacturing industry: rather than delivering a standardized commodity product at a fixed price, they provide unique and customer-specific solutions in a process of close interaction with their customers. The customers are usually firms themselves: a typical example of what we have in mind is a scenario in which an IT systems integrator like IBM provides IT solutions to a large enterprise like GM. The generated service value is often of highly intangible nature, which makes it hard to measure and virtually impossible to contract upon. The crucial role of customer participation is another distinct dimension of a service: because the provision of any service involves a transformation of the customer himself, the degree of customer participation directly affects the value of the service outcome. By learning from customer participation, the service firm also increases its stock of customer-specific knowledge which enables it to deliver higher service value to the same client in the future.

In this paper, we show that the aforementioned distinct dimensions of knowledge-based service industry can yield forms of competition that are vastly distinct from those known from manufacturing industry. In a typical manufacturing scenario, it is highly desirable for a producer to outpace all competitors in terms of production-relevant knowledge because the surplus that goes to the producer keeps increasing with the knowledge stock. As we will show, the opposite holds in a knowledge-based service industry. The continuous need to elicit customer participation in the value generation process, and the impossibility to agree on this participation ex-ante (because of incomplete contracts) makes it favorable for the service firm with the highest knowledge stock to share a portion of its production-relevant knowledge with competitors. By deliberately strengthening competitors, the firm can mitigate customer concerns about being held up in the future and can generate not only higher social welfare but also higher individual profits due to stronger customer participation. To the extent to which software source code embodies parts of the internal knowledge of an IT firm, we are therefore able to explain the commercial provision of open source software by leading IT service firms as a form of optimal knowledge sharing in knowledge-based service industry.

The perspective of our paper is related to, but different in several aspects, from the view taken in Shepard (1987) and Farrell and Gallini (1988) who show that a monopolist may have incentive to create competition in order to be able to commit to a low price or a high quality. First, their models are essentially static in the sense that they describe transactions in commodity goods whose value to the buyer is constant over time. This stands in sharp contrast to the service-centered perspective of our model where customers need to decide strategically which service provider they choose and how much effort they exert in the joint co-creation of value because their future payoffs are directly affected by these choices.

Second, unlike Shepard (1987), we assume an environment in which knowledge is tacit and not transferable to rival firms through licensing, for instance due to limited enforceability of intellectual property rights. As a consequence, in our model, the incumbent cannot use a fixed fee to extract the benefit that accrues to a rival firm from an increase in the latter’s
stock of knowledge. This situation is quite coherent with what happens when an incumbent makes a software available as open source.

Before we proceed to present our service-centered model, we briefly mention contributions from the literature that we find relevant and related to our work, but that did not fit into the discussion above. A very early work by Kuan (2001) has emphasized the aspect of consumer integration into production through open source projects; von Hippel and von Krogh (2003) have also made important points with respect to this user innovation aspect. Casadesus-Masanell and Ghemawat (2006) employ a “demand-side learning” parameter in their dynamic model of competition between an open source and a closed source product; this idea comes close to the concept of knowledge accumulation from user collaboration that we have in mind. However, unlike in their model, we will assume collaboration as costly to the user in order to model key strategic dimensions of a knowledge-based service exchange: our model captures B2B (Business to Business) transactions while their model captures B2C (Business to Consumers) transactions where consumers are not strategic.

The remainder of this paper is structured as follows: in section 2, we present our general model of knowledge-based service industry. We then dissect the general model into two illustrative special cases: section 3 analyzes a reduced model with only ex-ante investment, whereas section 4 takes ex-ante investment as exogenous and focuses on customer ex-post collaboration. Section 5 develops two important extensions to the model, and section 6 concludes.

2 A Model of Knowledge-Based Service Industry

2.1 Service Transactions and Service Value

In order to model service transactions, we first need to clarify along which dimensions we consider a knowledge-based service transaction to be different from a commodity good purchase. We adopt the view of Zeithaml, Parasuraman, and Berry (1985) who propose the following four defining characteristics of a service as opposed to a commodity transaction:

1. Intangibility

2. Heterogeneity (inability to standardize)

3. Inseparability (of production and consumption)

4. Perishability (inability to inventoried).

These properties stand in sharp contrast to typical Industrial Organization models of commodity industry (in particular, models of B2C transactions) which assume that a producer first produces a standardized, tangible and storable commodity which is subsequently sold in a separate step to mass consumers. Intangibility and perishability will make contracting on
service value difficult. The notion of heterogeneity fits well the B2B service transactions that we have in mind. But the in our opinion most relevant and most unique aspect of a service transaction is the inseparability of consumption and production. It implies that service value is co-created by the consumer and the service firm, which leads us to adopt the following definition of a service:

**Definition.** *(Fitzsimmons and Fitzsimmons, 2001): A time-perishable, intangible experience performed for a customer acting in the role of co-producer.*

The lack of standardization and the inseparability of consumption and production make it obvious that a production function (which captures produced quantities of a standardized commodity good) is of little use for quantifying production in service industry. It seems more reasonable to capture service production by a function $V(\cdot)$ that describes the *value* that is co-created by the consumer and the service firm from a given quantity of input factors.

We postulate that in the context of knowledge-based service industry, the created service value is a function of three input factors:

$$V: \mathbb{R}^3 \rightarrow \mathbb{R}$$

$$V(K, s, x)$$ (1)

The key input factor supplied by the service firm is its relevant *customer-specific knowledge stock* $K$. Customer-specific knowledge enables the service firm to understand the customer’s problem in more detail and to deliver a better solution which increases service value. We postpone a more detailed discussion of knowledge stocks to the next subsection. The two other factors capture the customer’s effort in collaborating towards a successful service outcome. We distinguish between *ex-ante investment* $s$ before the service contract is signed, and *ex-post collaboration* $x$ after the contract has been signed:

By *ex-ante investment* $s$, we mean the size of project-related irreversible investment that the customer makes before actually soliciting an offer from service firms. Basically, what we have in mind is that the customer, for instance a big firm, decides at the beginning of each period her overall business plan and the strategic role of the IT service in this plan. This in turn determines the size of the IT service that she will ask service providers to deliver, and the primary budget of the project. This decision on the level of ex-ante investment has some irrevocable element since revising it requires revising the entire business plan.

By *ex-post collaboration level* $x$, we mean the intensity and quality of tasks and practices that are performed by the customer at her own cost, but that positively affect the value of the service. Examples of such tasks in an IT service transaction environment are: generating a robust customer-side requirement analysis in the run-up for project specification; providing the service firm with documentation on internal processes and business organization; ensuring the disposability of customer staff for coordination with the service firm (for activities such as

\footnote{In fact, we will assume that the value created by a service firm is not contractible.}
specification refinement, testing and the preparation of migration); supporting maintenance
issues that can only be resolved jointly (e.g., bug regression); and promoting high levels of
user skills and IT awareness among end-users and management.

As the above discussion illustrates, the dimensions of customer investment and customer
collaboration comprise of many aspects that are difficult to measure objectively and thus
impossible to contract upon. We therefore assume that \( s \) and \( x \) are observable but not
contractible as in the incomplete contracting literature (Grossman and Hart, 1986; Hart and
Moore, 1988), and as a consequence, the service value is not contractible either. Furthermore,
we assume that the client cannot commit in advance to future choices of \( s \) and \( x \) and so the
client and a firm cannot sign a long-term contract.

In order to focus on the client’s choice of ex-ante investment and collaboration level, we
deliberately abstract from the moral hazard problem on the part of service providers, which
arises because \( V(K, s, x) \) is not contractible. More precisely, some papers on reputation such
an agent’s incentive to work (or shirk) when the quality of the service (or product) that
he produces is not contractible and the client should pay the price before the service is
produced (or before he consumes the product). Our \( V(K, s, x) \) can be interpreted as the
value generated either when reputational concern allows the service provider to overcome his
incentive problem or when he cannot overcome the problem. In other words, we focus on
the client’s choice of collaboration for given degree of incentive problem on the part of the
service providers.

Both ex-ante investment and ex-post collaboration are costly for the customer. We model
the customer’s cost of ex-ante investment \( s \) and ex-post collaboration \( x \) by introducing cost
functions \( C_s(s) \) and \( C_x(x) \), where we assume that these functions are convex and twice
differentiable and satisfy \( C_s(0) = C_x(0) = 0 \), \( \lim_{s \to 0} C_s'(s) = \lim_{x \to 0} C_x'(x) = 0 \), and \( C_s'' > 0 \), \( C_x'' > 0 \). With respect to the cost incurred by the service provider, we postulate that the
cost to deliver the service is constant with respect to \( K \), \( s \) and \( x \). For the sake of simplified
notation, we will set it equal to zero.

Finally, we assume that the service value function \( V(K, s, x) \) has the usual analytical
properties of a neoclassical production function: it is differentiable twice, has positive but
diminishing returns to \( K \), \( s \) and \( x \), and all cross-derivatives are positive,

\[
\frac{\partial^2}{\partial s \partial K} V(K, s, x) > 0, \quad \frac{\partial^2}{\partial x \partial K} V(K, s, x) > 0, \quad \frac{\partial^2}{\partial s \partial x} V(K, s, x) > 0.
\]

In other words, higher specific knowledge \( K \) allows for higher returns to ex-ante investment
and customer collaboration, and vice versa. We also assume that no value can be produced
without knowledge: \( \lim_{K \to 0} V(K, s, x) = 0 \).
2.2 Knowledge Stocks and Knowledge Accumulation

The final ingredient to our model of knowledge-based service industry is a notion of learning effects from repeated service transactions: whenever a knowledge-based service firm delivers a service, its interaction with the client allows it to privately observe information that increases the service firm’s knowledge stock about this particular customer. A higher knowledge stock, in turn, increases the service value that the firm can generate in future service transactions with the same customer. For example, in the context of B2B IT service transactions a service firm may be able to learn about ways of integrating its solutions with the existing IT architecture of its client, about the structure of its client’s business processes, or about unexploited opportunities to improve the client’s manufacturing process with the help of information technology. Such knowledge is valuable because it allows the IT firm to deliver more tailor-made solutions in the future.

Clearly, the amount of learning depends on how much information the customer is willing to disclose, which is closely related to the customer’s chosen level of participation before and during the service transaction. For our model of knowledge-based service industry we thus assume that the specific knowledge stock of a service firm that performs a service transaction for a customer evolves according to

\[ K_{t+1} = K_t + k(K_t, s_t, x_t) \]  

(2)

where \( k(K, s, x) \) is a learning function that is increasing in all three parameters, concave and satisfies \( \lim_{K \to 0} k(K, s, x) = 0 \) for all \( s, x \). We postulate that a higher knowledge stock of the firm increases the marginal learning from both customer collaboration and from project size:

\[ \frac{\partial^2 k}{\partial K \partial x} > 0, \quad \frac{\partial^2 k}{\partial K \partial s} > 0 \quad \text{and} \quad \frac{\partial^2 k}{\partial s \partial x} > 0. \]

Finally, it is important to emphasize that for most knowledge-based services, the knowledge stock \( K_t \) is of tacit nature and can’t be transferred to rival firms by licensing. We will adopt this assumption throughout our model. Often the reason for this restriction lies in the limited availability or enforceability of intellectual property rights: for example, IT service providers who have detailed knowledge of possible software integration paths in a particular client’s IT infrastructure can not claim any kind of intellectual property rights on such knowledge. As a consequence, a service firm cannot use a fixed fee to extract the benefit that accrues to a rival firm from an increase in the latter’s stock of knowledge. Instead, a firm can only decide whether to keep its knowledge private or to give it away for free. This situation is quite coherent with what happens when an incumbent makes a software available under an open source license.
2.3 General Model of Knowledge-Based Service Transactions

We now have everything in place to write down a dynamic model of knowledge-based service industry: There are \( N \geq 2 \) service firms that are competing to provide a knowledge-based service to one single customer. The firms are endowed with initial knowledge stocks \( K_1^1, K_2^1, \ldots, K_N^1 \). Without loss in generality, we shall assume that firms are arranged such that \( K_1^1 \geq K_2^1 \geq \cdots \geq K_N^1 \). Time is discrete, \( t = \{1, 2, \ldots\} \). Every period of the model comprises of a game in which the players play the following steps:

1. the customer chooses the size of the project by making an ex-ante investment of \( s_t \) at a cost of \( C_s(s_t) \); these cost become sunk immediately

2. the customer receives bids \( p_i(s_t) \) from all firms \( i \in \{1, 2, \ldots, N\} \) who offer to execute the project; the customer chooses the firm that leaves her the highest surplus, taking into account how the chosen service firm’s learning will increase or reduce her surplus in the following periods. In case the customer is indifferent between firms, we assume that she chooses the incumbent (i.e., the firm with the highest knowledge stock).

3. the customer provides on-project collaboration effort \( x_t^i \) and incurs the corresponding cost \( C_x(x_t^i) \)

4. at the end of the period, the service value \( V(K_t^i, s_t, x_t^i) \) materializes, firms’ knowledge stocks update according to the corresponding equations of motion, and the customer obtains a total period surplus of

\[
S_t(K_t^i, s_t, x_t^i) = V(K_t^i, s_t, x_t^i) - C_s(s_t) - C_x(x_t^i) - p_i(s_t) \tag{3}
\]

Firms maximize the net present value of overall profits, whereas the customer maximizes the net present value of overall customer surplus; both the customer and the firm discount the future by the same discount factor \( \delta < 1 \).

We believe that this model captures in a quite general manner key aspects of knowledge-based services, i.e. the co-creation of value between customers and service firms, and the existence of a dynamic learning channel. Although this general model with infinite horizon could be solved with recursive techniques, the intuition for its properties is conveyed better by restricting the model to two special cases: in what follows, we will solve two two-period versions of the general model where we focus on only one of the two customer choice variables at a time: in the first model, we will solve for the customer’s optimal choice of ex-ante investment \( s_t \) under different knowledge sharing policies, whereas our second model takes investment size \( s_t \) as exogenously fixed and completes the picture by analyzing the customer’s optimal choice of ex-post collaboration \( x_t \).
3 Knowledge Sharing and Ex-ante Investment

In this section, we will focus on understanding the impact of various knowledge sharing policies on customer ex-ante investment and service firm profits. We thus choose to completely abstract from the aspect of ex-post customer collaboration. This can be attained easily by assuming that the learning function \( k(K, s, x) \) does not depend explicitly on the collaboration effort \( x \),

\[
k(K, s, x) = k_s(K, s).
\]

With this simplification, the client’s choice of ex-post collaboration \( x \) in one period will leave the service offers of future periods unaffected, and in equilibrium the client thus simply chooses the optimal static collaboration level \( \bar{x}^i = \bar{x}^i(K^i, s) \) with firm \( i \) according to the first order condition \( \frac{\partial}{\partial x} V(K^i, s, \bar{x}^i) = C'_x(\bar{x}^i) \). We can therefore transform the problem and conveniently hide all terms related to ex-post collaboration \( x^i \) in a new definition of the service value function\(^2\):

\[
V(K^i, s) \equiv V(K^i, s, \bar{x}^i(K^i, s)) - C_x(\bar{x}^i(K^i, s)) \tag{4}
\]

In what follows, we compare for a given level of the incumbent’s initial knowledge stock \( K_1 \) the equilibrium ex-ante investments, customer surplus and profits of three games \( \{\Gamma_C, \Gamma_O, \Gamma_R\} \). Each game corresponds to the incumbent’s adoption of one of the following three knowledge sharing policies: closed source (C), open source (O) and reciprocal open source (R). We precise on the definition and the economic meaning of these policies:

**Closed Source (C):** If the incumbent selects closed source, he retains exclusivity over his customer-specific knowledge. This makes him the only firm that is able to offer positive service value. We capture this by assuming that the initial knowledge stock of all entrants is zero, \( K_1^E = 0 \). Moreover, under closed source only the firm \( i \) that is chosen to provide the service can realize positive learning effects from the co-creation of service value with the customer:

\[
K_{t+1}^j = \begin{cases} 
K_t^j + k_s(K_t^i, s_t) & \text{for the chosen firm } j = i \\
K_t^j & \text{for all other firms } j \neq i
\end{cases} \tag{5}
\]

In other words, the choice of a closed source license allows the incumbent to keep all his static knowledge stock and all dynamic knowledge gains private.

**Open Source (O):** If the incumbent adopts open source, he gives up his monopoly position over the customer: by releasing (some of) the customer-specific software which it uses to deliver the service under an open source license, the incumbent makes a portion of his knowledge stock freely available to entrants. This creates competition because the new

\(^2\)Note that, by the nature of the transformation, all our assumptions regarding the properties of \( V \) and its derivatives carry over to \( V \).
entrants can use this knowledge to bid on providing a service to the same customer. In the model, we capture this by assuming that the release of the software as open source means that the incumbent shares a positive fraction\(^3\) of his initial knowledge stock with entrants, so \(0 < K_i^E < K_i^I\).

The mere adoption of an open source license does not imply that dynamic knowledge gains are also shared: for example, if code is made available under a BSD license, there is no requirement for products that modify or improve the original code to be released again as open source. Similarly, firms that resort to dual-licensing (see Comino and Manenti, 2007) are not committed to sharing any dynamic knowledge gains because they typically reserve the right to ship their own improvements under proprietary licensing terms. Thus we assume that knowledge stocks evolve under the same law of motion as under closed source and follow equation (5).

**Reciprocal Open Source (R):** As a third knowledge-sharing policy, we consider the case in which the incumbent publishes his software exclusively under a reciprocal open source license such as the GNU General Public License. Such reciprocal open source licenses state that licensees must share all improvements to the software under the license terms of the original code (and thus make them open source). Many commercial software firms that have released software under such license terms have even taken additional measures to further underline their commitment to keeping their development process open for everyone, for example by transferring the oversight of the development process to a non-profit foundation.

We interpret such choices on the side of the incumbent as a commitment to sharing not only static knowledge stocks but also all future dynamic knowledge gains. Moreover, due to the reciprocal terms of the license any entrant will be equally obliged to share all dynamic increases of his knowledge stock. We will model this situation by postulating that all dynamic changes to the knowledge stock of the chosen firm \(i\) will equally accrue to all its competitors,

\[
K_{i+1}^j = K_i^j + k_s(K_i^i, s_t) \quad \text{for all firms } j \in \{1, \ldots, N\}
\]  

(6)

Note that for the sake of simplicity we have assumed that the rival firms’ gain is equal to the full amount of the dynamic knowledge gain of the service firm that was actually chosen for the service contract.

We will now solve the three games \(\Gamma_C, \Gamma_O, \Gamma_R\) which have the following structure: First, the initial knowledge stocks of entrants is set according to the static knowledge sharing policy of

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\(^3\)We assume that the incumbent still retains an advantage over the entrants because knowledge sharing via open source is not perfect. For example, some relevant parts of the code might be rather obfuscated and may be well-understood only by the incumbent’s developers, or the incumbent’s developers hold additional private knowledge that is not reflected in the source code. In an extension of our analysis, we also discuss what determines the fraction of knowledge stock that the incumbent would optimally want to share.
the corresponding game, i.e.

\[ K_1^2 = K_1^3 = \cdots = K_1^N = \begin{cases} 0 & \text{for } \Gamma_C \\ K_1^F & \text{for } \Gamma_O, \Gamma_R \end{cases} \]

Next, the firms compete for two periods \( t = \{1, 2\} \) to deliver a knowledge-based service to the customer. The structure and timing of each period follows our general service model\(^4\):

1. the customer chooses ex-ante collaboration \( s_t \) and pays \( C_s(s_t) \)
2. all service firms quote their prices \( p^i(s_t) \)
3. the customer chooses the firm that offers the best overall surplus, taking into account how her choice affects future periods
4. a service value of \( \mathcal{V}(K_i^t, s_t) \) is produced and knowledge stocks are updated according to the dynamic knowledge sharing policy of the game, i.e. equation (5) for \( \Gamma_C, \Gamma_O \) and equation (6) for \( \Gamma_R \).

### 3.1 Benchmark: Planner’s Solution

Before we establish the equilibria of the service transaction games under closed source and open source, we first pin down the socially optimal choices as a benchmark: which levels of ex-ante investment should a social planner choose whose exclusive concern is to maximize social welfare? Clearly, the highest social welfare is attained if the planner allocates the service transaction to the most knowledgeable service provider; hence, the planner solves the problem

\[
\max_{s_1^*, s_2^*} \mathcal{V}(K_i^t, s_t) - C_s(s_1) + \delta \left[ \mathcal{V}(K_i^t + k_s(K_i^t, s_1), s_2) - C_s(s_2) \right]
\]

We thus find that the socially optimal investments \( s_1^* \) and \( s_2^* \) are uniquely pinned down by the first-order conditions

\[
\frac{\partial \mathcal{V}}{\partial s} \bigg|_{(K_i^t, s_t^*)} = C_s'(s_2^*) \text{ and } \quad \frac{\partial \mathcal{V}}{\partial s} \bigg|_{(K_i^t, s_1^*)} + \delta \frac{\partial \mathcal{V}}{\partial K} \bigg|_{(K_i^t + k_s(K_i^t, s_1^*), s_2^*)} \cdot \frac{\partial k_s}{\partial s} \bigg|_{(K_i^t, s_1^*)} = C_s'(s_1^*)
\]

The meaning of these equations is quite intuitive: whilst in the second period the planner simply chooses the statically optimal amount of ex-ante investment, in the first period he additionally takes dynamic gains from knowledge accumulation into account and thus selects an investment level higher than the statically optimal one.

\(^4\)in its reduced form with static ex-post collaboration effort
3.2 Game $\Gamma_C$: Closed Source

We can now proceed to solve the game $\Gamma_C$ of closed source service provision. The incumbent firm will enjoy monopoly power over the customer since all entrants’ knowledge stocks (and thus ability to produce value) are zero. The incumbent will win both periods of the game, and entrants will never acquire any knowledge. But monopoly power will enable the incumbent to extract all surplus: in the second period, the incumbent will charge a price of

$$p_C^2(s_C^2) = \mathcal{V}(K_1^I + k_s(K_1^I, s_C^1), s_C^2)$$

and leave the customer with a second period surplus of $-C_s(s_C^2)$. Hence, the customer will optimally abstain from making any irreversible ex-ante investment before writing a contract with the incumbent service firm: $s_C^2 = 0$. Going backwards, the same logic also applies to the first period, and we have

**Proposition 1.** Under closed source, there is zero ex-ante investment in both periods: $s_C^1 = s_C^2 = 0$. The total surplus that is generated amounts to $\mathcal{V}(K_1^I, 0) + \delta \cdot \mathcal{V}(K_1^I + k(K_1^I, 0), 0)$. All surplus accrues to the incumbent.

We see that the combination of monopolistic market power with lack of commitment not to hold up the customer for her irreversible ex-ante investments eradicates all incentives to make such investment in the first place. Thus, relative to the planner’s solution, social welfare will be lost: this effect is most pronounced for highly customized services whose value $\mathcal{V}(K, s)$ increases steeply with $s$, and it is least relevant for transactions like commodity purchases that do not derive much additional value from irreversible customer ex-ante investment.

3.3 Game $\Gamma_O$: Open Source (without commitment)

Under open source, the incumbent shares some of his initial knowledge stock with all entrants: all entrants obtain free access to the source code of the software which endows them with a positive initial knowledge stock $K_1^E < K_1^I$ and enables them to enter the market. In this section, we solve the model assuming that the incumbent chooses a non-reciprocal open source licensing model (such as BSD license) and that neither the incumbent nor any of the entrants can commit to sharing their dynamic knowledge gains with competitors.

Again, we solve the game by backward induction. For a given ex-ante investment $s_1$ in the first period, knowledge stocks in the second depend on which firm was awarded the service project in the first period. We therefore distinguish two cases:

3.3.1 Case 1: Second period if the incumbent was awarded the first period

Suppose that the incumbent has run the project in the previous period. Then learning effects increase the knowledge stock of the incumbent to $K_1^I + k_s(K_1^I, s_1)$ whereas all entrants’
knowledge stocks remain at $K_{I}^{E}$. Since any of the entrants can at most offer the entire service value that would be generated, the incumbent can charge a price of up to

$$p_{2,I}^I(s_2) = V(K_{I}^{I} + k_s(K_{I}^{I}, s_1), s_2) - V(K_{I}^{I}, s_2)$$

without losing the customer to an entrant. We will use the subscript $2,I$ to represent the second-period payoffs when when the incumbent was awarded the project in period one. For a given level of second-period investment $s_2$, the customer surplus and each firm’s profit in period two reads

$$S_{2,I} = V(K_{E}^{I}, s_2) - C_s(s_2)$$
$$\Pi_{2,I}^I = V(K_{1}^{I} + k_s(K_{1}^{I}, s_1), s_2) - V(K_{E}^{I}, s_2)$$
$$\Pi_{2,I}^E = 0 \text{ (for all entrant firms)}$$

Let us denote the equilibrium level of ex-ante investment under open source in the second period when the incumbent was awarded the first period as $s_{2,I}^{O}$. It maximizes customer surplus and is thus determined by the first-order condition

$$C'_s(s_{2,I}^{O}) = \frac{\partial V}{\partial s} \bigg|_{(K_{E}^{I}, s_{2,I}^{O})}$$ (10)

We note that the second period ex-ante investment $s_{2,I}^{O}$ is independent of the previous period collaboration level $s_1$. Moreover, it is easy to see that $s_{2,I}^{O}$ is strictly greater than zero (which was the level $s_{2,I}^{C}$ attained under closed source) but falls short of the socially optimal level $s_2^*$.

### 3.3.2 Case 2: Second period if an entrant was awarded the first period

If the customer chooses an entrant in the first period as her service provider, the incumbent’s knowledge stock in period two will remain at its initial level $K_{I}^{I}$ whereas the knowledge stock of the selected entrant will be augmented by learning effects from the first period service transaction: $K_{E}^{I} = K_{E}^{I} + k_s(K_{E}^{I}, s_1)$. The outcome of the second period competition now depends to a considerable extent on whether the entrant’s dynamic learning effect is sufficiently large to surpass the incumbent in terms of knowledge stock, i.e. whether

$$K_{E}^{I} + k_s(K_{I}^{I}, s_1) > K_{I}^{I}.$$ (11)

We will refer to this situation as substantial learning. If learning is substantial, the entrant firm will win the service contract in the second period as well and it will make positive profits; otherwise, the incumbent wins the second period contract and extracts the part of customer surplus that is not protected by the customer’s outside option of choosing the entrant. Let us denote as $H$ the firm $\in \{E,I\}$ with the higher second period knowledge stock, and as $L$ the firm with the second-highest second period knowledge stock, both conditional on the
first period being run by the entrant:

\[ K_{2H}^L \equiv \max\{K_1^I, K_1^E + k_s(K_1^E, s_1)\} \]
\[ K_{2L}^L \equiv \min\{K_1^I, K_1^E + k_s(K_1^E, s_1)\} \]

Then, by the same arguments as in the previous case, the most knowledgeable firm \( H \) (which can now be either the incumbent or the entrant) will win the contract in period two, will charge a price of

\[ p_{2,E}(s_2) = \mathcal{V}(K_{2H}^H, s_2) - \mathcal{V}(K_{2L}^L, s_2) \]

and leave the customer with a surplus of

\[ S_{2,E} = \mathcal{V}(K_{2L}^L, s_2) - C_s(s_2) \]

such that the customer’s optimal second period choice of ex-ante investment \( s_{2,E}^O \) will be determined by the first-order equation

\[ \frac{\partial \mathcal{V}}{\partial s} \bigg|_{(K_{2L}^L, s_{2,E}^O)} = C_s'(s_{2,E}^O) \]

We observe that, since \( K_{2L}^L > K_1^E \) and \( \frac{\partial^2 \mathcal{V}}{\partial K^2} > 0 \), the customer’s marginal gains from ex-ante investment are higher if an entrant rather than the incumbent was chosen for the first period. Thus, awarding the project to the entrant in the first period will induce strictly higher second period ex-ante investment than if the first period project is awarded to the incumbent. Nevertheless, because \( K_{2L}^L < K_1^I + k_s(K_1^I, s_1) \) the amount of ex-ante investment in both cases falls short of the socially optimal amount that we have established in the planner’s benchmark. In addition, we show in appendix 6 that the higher amount of ex-ante investment under the entrant also generates a higher surplus for the customer in period two than if the customer had chosen the more knowledgeable incumbent in period one.

Summarizing the results of the game’s second period, we have:

**Observation** In period two, ex-ante investment is lower than the social optimum. The underinvestment is most severe when the first period project was awarded to the incumbent, and less severe when when the entrant was chosen in the first period. There is an intrinsic conflict between choosing the right service firm in period one and choosing an efficient level of ex-ante investment in period two.

Before moving on to the first period, it will be useful to clarify already at this stage how the equilibrium ex-ante investment level in period two depends on the choice of \( s_1 \). Note that the customer’s incentive to invest ex-ante in the second period depends only on the level of \( K_1^E \) (if the incumbent was awarded the first period) or the level of \( K_{2L}^L \) (if an entrant was awarded the first period). Only the latter could possibly be a function of \( s_1 \) because \( K_{2L}^L \)
increases in \( s_1 \) if learning is non-substantial. Therefore, we note that the equilibrium choice of \( s_2 \) does not depend on \( s_1 \) if the entrant’s learning is substantial or if the incumbent wins the first period. Otherwise, the equilibrium level of \( s_2 \) will be increasing in \( s_1 \).

### 3.3.3 First Period

In period one, if the incumbent wants to win the contract, he can always win it. However, it may be optimal not to win the contract. We first conduct the analysis under the assumption that the incumbent wins the contract and then study when it is optimal for him to actually do so.

Solving backwards, let us for now take \( s_1 \) as given and analyze the prices \( p^E_1(s_1) \) that the competing service firms will charge in equilibrium as they attempt to win the business of the customer in the first period. If learning is not substantial \((K^E_1 + k(K^E_1, s_1) < K^I_1)\), the best offer an entrant can make is a price of \( p^E_1 = 0 \). If learning is substantial, an entrant can pledge the second period profit that he can realize if he is chosen as the service provider. Thus, entrants will offer to run the project in the first period for a price of

\[
p^E_1(s_1) = -\left[ \delta \cdot \max\{0, V(K^E_1 + k(K^E_1, s_1), s^O_2, E) - V(K^I_1, s^O_2, E) \} \right]
\]

In order to win the customer in period one, the incumbent has not only to match this offer but he also needs to compensate the customer for the partial loss of her second period surplus that she will incur if she chooses the incumbent in period one:

\[
p^I_1(s_1) = V(K^I_1, s_1) - V(K^E_1, x_1) - p^E_1 - \delta (S^O_2 - S^I_2)
\]

As a consequence, the overall surplus that accrues to the customer under open source when he chooses the incumbent is equal to

\[
S^O_{\text{total}} = V(K^I_1, s_1) - p^I_1(s_1) - C_s(s_1) + \delta \cdot S^I_2
\]

which after substituting expressions and further simplification\(^5\) becomes

\[
S^O_{\text{total}} = V(K^E_1, s_1) - C_s(s_1) + \delta \left[ V(K^E_1 + k_s(K^E_1, s_1), s^O_2, E) - C_s(s^O_2, E) \right]
\]

Basically, the customer’s total surplus is given by the payoff that she can achieve when she has free access to the entrant’s knowledge. A minor qualification is that \( s^O_{2,E}(s_1) \) maximizes \( V(K^E_1 + k_s(K^E_1, s_1), s_2) - C_s(s_2) \) only when the innovation is not substantial.

Finally we can solve the first step of the game: the customer chooses the level of ex-ante investment \( s_1 \) in order to maximize her overall surplus \( S^O_{\text{total}} \). The optimal level \( s^O_1 \) of ex-ante

\(^5\)In particular, we exploit the fact that the entrant’s second period profits and the second period customer surplus under the entrant must sum to \( V(K^E_1 + k_s(K^E_1, s_1), s^O_2, E) - C_s(s^O_2, E) \).
investment in period one is therefore characterized by the first-order condition\(^6\)

\[ \frac{\partial V}{\partial s} \bigg|_{(K_1^E, s_1^O)} + \delta \left[ \frac{\partial V}{\partial K} \bigg|_{(K_1^E + k_s(K_1^E, s_1^O), s_2^O_E)} \cdot \frac{\partial k_s}{\partial s} \bigg|_{(K_1^E + k_s(K_1^E, s_1^O), s_2^O_E)} \right] = C_s'(s_1^O) \]

(13)

This equation has a deeper economic interpretation: It shows that ex-ante investment in the first period generates two sources of benefits to the customer. First, it generates the immediate benefit of increasing the value of the period one service and the customer appropriates a part of it. Second, it increases the value that an entrant can produce in period two from accumulating customer-specific knowledge if the same firm was also chosen for the first period. Since this raises the value of the outside option of the customer, the customer benefits from it. In other words, the customer can sell the right to learn about her service needs.

We can now compare the equilibrium value of period one investment size \(s_1^O\) under open source to first-best benchmark \(s_1^*\) which yields the following insight:

**Proposition 2.** In the game \(\Gamma_O\) of ex-ante investment under open source without commitment, the customer’s ex-ante investment in period one is strictly higher than under closed source, i.e. \(s_1^O > s_1^C\). The position of \(s_1^O\) relative to the socially optimal level \(s_1^*\) is undetermined: depending on the functional forms of \(V(\cdot, \cdot)\), of \(k_s(\cdot, \cdot)\) and the magnitude of the discount factor \(\delta\), \(s_1^O\) can be inefficiently low, efficient or excessive. For sufficiently small discount factor \(\delta\), \(s_1^O\) will always be inefficiently low.

**Proof.** see appendix. \(\square\)

It is interesting to note that open source can induce socially excessive ex-ante investment in the first period. This occurs because the marginal impact of an increase of knowledge stock on productivity is higher for the entrant than for the incumbent due to diminishing returns to knowledge.

Finally, we calculate the profits for the incumbent: the incumbent’s payoff is the total value it produces minus the total value that entrants who win the customer for two periods would generate, so

\[ \Pi_{\text{total}}^{I,O} = V(K_1^I, s_1^O) - V(K_1^E, s_1^O) + \delta \left[ C_s(s_2^E) - C_s(s_2^I) \right] + \delta \left[ V(K_1^I + k_s(K_1^I, s_1^O), s_2^I) - V(K_1^E + k_s(K_1^E, s_1), s_2^O_E) \right] \]

The term related to cost, \(C_s(s_2^E) - C_s(s_2^I)\), represents the customer’s lower investment cost in period two in case the incumbent is selected in period 1. Note that the incumbent does not bear the customer’s period one cost of ex-ante investment since it is already sunk.

\(^6\)Note that from the envelope theorem, we can neglect the indirect effect through the change \(s_{1,E}^O\) when learning is not substantial: furthermore, when learning is substantial, \(s_{1,E}^O\) does not depend on \(s_1\).
3.3.4 When does the incumbent want to win the first period?

Up to now we have assumed that the incumbent wants to win the period one contract. However, if the initial gap in terms of knowledge stock is large and learning is non-substantial, the incumbent may be better off letting an entrant win the period one contract in order to boost the customer’s period two ex-ante investment. Since there is free entry under open source, an entrant’s overall profit is always zero whereas the customer gets all the benefit from having free access to the entrant’s technology. Thus, the customer’s payoff for a given \( s_1 \) is the same as in the previous subsection and the equilibrium choice of \( s_1 \) remains unchanged.

We can therefore directly compare the incumbent’s payoff conditional on winning the period one contract and his payoff conditional on losing it: the incumbent will choose to lose the first period contract whenever his payoff conditional on the entrant serving the first period contract,

\[
\Pi_{I,O}^{\text{onlyperiod2}} = \delta \left\{ V(K^I_1, s^O_2, E) - V(K^E_1 + k_s(K^I_1, s^O_1), s^O_2, E) \right\},
\]

exceeds the overall profits \( \Pi_{I,O}^{\text{total}} \) from serving the customer for both periods.

In order to better understand the conditions under which it remains optimal for the incumbent to win the first period contract, let us consider just for a moment a static one-period version of the game with fixed knowledge stocks \( (K^I, K^E, \ldots) \): in a static model, the customer’s optimal investment choice \( \bar{s} \) will be pinned down by the first-order condition

\[
\frac{\partial V}{\partial s} \bigg|_{(K^E, \bar{s})} = C'(\bar{s})
\]

which implies that \( \bar{s} \) is an increasing function of the entrants’ knowledge stock: \( \bar{s} = \bar{s}(K^E) \). The incumbent’s profit function will be

\[
\Pi^I(K^E) = V(K^I, \bar{s}(K^E)) - V(K^E, \bar{s}(K^E)).
\]

Clearly, if \( K^E \) is sufficiently close to \( K^I \), profits will be decreasing in \( K^E \): as the knowledge gap between the incumbent and entrants becomes smaller, the incumbent’s profits fall towards zero. However, if entrants have substantially smaller knowledge stocks than the incumbent (i.e., \( K^E \ll K^I \)), profits can be increasing in \( K^E \) because higher entrant knowledge will induce higher customer investment \( \bar{s}(K^E) \). Assume that \( \Pi^I(K^E) \) is strictly concave and reaches its maximum for some value \( K^{E*} \in [0, K^I] \). Going back to our dynamic model, \( K^E \geq K^{E*} \) will then be a sufficient condition under which the incumbent will never yield the first period to an entrant: to see this, note that the incumbent’s second period profit in the dynamic model depends on \( K^E \) exactly like the profit function in a static game, and thus the incumbent’s second period (and overall) profits would decrease if the entrant were to gain further knowledge.

The fact that the incumbent may find it optimal not to win the first period also suggests
that knowledge-based service firms may use market segmentation as a mechanism to induce
customer investment. Although we deliberately focused on the case of one buyer, it is easy
to extend the result to a setting with two buyers where the knowledge accumulated from one
customer is partially applicable to another and vice versa. In this situation, even though the
incumbent’s production technology has constant return to scale and hence he can win both
customers if he wants, the incumbent may deliberately leave a customer to a competitor in
order to boost customer investment.

3.4 Game $Γ_R$: Reciprocal Open Source

In this section, we finally explore the equilibrium of the game $Γ_R$ in which the incumbent can
commit, for example by using a reciprocal open source license, to incorporate all dynamic
knowledge gains from the provision of the service in his open source software. We again solve
backwards, starting with the second period. Since all firms are committed to sharing all their
knowledge gains, the incumbent will always have a larger knowledge stock than any entrant,
irrespectively of which firm was chosen in period one. Hence, the incumbent always wins
both periods. For the sake of brevity, we restrict our discussion to this particular case. The
full analysis of all subgames of game $Γ_R$ (as needed to establish the first period price charged
by the incumbent) can be found in the appendix.

If the incumbent was selected in the first period, his second period knowledge stock
will be $K^I_2 = K^I_1 + k_s(K^I_1, s_1)$ whereas all entrants will have knowledge stocks of $K^E_2 =
K^E_1 + k_s(K^I_1, s_1)$. Thus, the incumbent firm will always win the second period and charge a
price of

$$p^R_{2,I}(s_2) = V(K^I_1 + k_s(K^I_1, s_1), s_2) - V(K^E_1 + k_s(K^I_1, s_1), s_2)$$

which leaves the customer with a surplus of

$$S^R_{2,I} = V(K^E_1 + k_s(K^I_1, s_1), s_2) - C_s(s_2).$$

The corresponding first-order condition for the customer’s second period’s choice of ex-ante
investment $s^R_{2,I}$ therefore is

$$\frac{\partial V}{\partial s} \bigg|_{(K^E_1 + k_s(K^I_1, s_1), s^R_{2,I})} = C'_s(s^R_{2,I})$$

which immediately unveils the following: under reciprocal open source, ex-ante investment $s^R_{2,I}$ in period two is greater than ex-ante investment $s^O_{2,I}$ under open source without commit-
ment, and it is always increasing in the first period investment level $s_1$. The reason for this
is that dynamic knowledge gains are now shared among all firms which raises the customer’s
outside options in period two. Thus, much of the additional second period surplus from
dynamic knowledge accumulation accrues to the customer. This is to be contrasted with the
equilibrium of the game $\Gamma_O$ of open source without commitment, where all the additional second period surplus from dynamic knowledge accumulation is extracted by the service firm.

Going back to the first period, the incumbent will charge a price of

$$p_1(s_1) = V(K^I_1, s_1) - V(K^E_1, s_1) + \delta [S^R_{2,I} - S^R_{2,E}].$$

We see that the incumbent can charge a premium over the immediate value offered by the entrant because the second period customer surplus is higher if the incumbent obtains the first period due to superior learning effects.

The customer will choose a level of first period ex-ante investment $s^R_1$ that maximizes her overall surplus,

$$S^R_{total}(s_1) = V(K^E_1, s_1) - C_s(s_1) + \delta [V(K^E_1 + k_s(K^E_1, s_1), s^R_{2,E}) - C_s(s^R_{2,E})],$$

so her optimal choice $s^R_1$ will satisfy the following first-order condition:

$$\frac{\partial V}{\partial s} \bigg|_{(K^E_1, s^R_1)} + \delta \frac{\partial V}{\partial k_s} \bigg|_{(K^E_1 + k_s(K^E_1, s^R_{1}), s^R_{2,I})} \cdot \frac{\partial k_s}{\partial s} \bigg|_{(K^E_1, s^R_1)} = C'_s(s^R_1).$$

Remarkably, this equation is almost identical to our result for the first-order condition under open source without commitment: the only difference is that the term in $\frac{\partial V}{\partial k_s}$ is now evaluated at a strictly higher second period investment level $s^R_{2,I} > s^O_{2,I}$. Comparing this finding with our previous results yields immediately the following insight:

**Proposition 3.** In the game $\Gamma_R$ of ex-ante investment under reciprocal open source, the incumbent always wins both periods. The first period ex-ante investment $s^R_1$ is always higher than the corresponding first period ex-ante investment $s^O_1$ under open source without commitment; its position relative to the benchmark level $s^*_1$ is undetermined, i.e. $s^R_1$ can be inefficiently low, efficient or excessive. Second period investment $s^R_{2,I}$ under reciprocal open source is strictly higher than under open source without commitment but it falls short of the efficient level $s^*_2$.

**Proof.** It follows immediately from comparing the relevant first-order conditions. \(\square\)

Finally, we can calculate the profits of the incumbent under reciprocal open source. We find:

$$\Pi^{I,R}_{total} = V(K^I_1, s^R_1) - V(K^E_1, s^R_1)$$
$$+ \delta \left[ V(K^I_1 + k_s(K^I_1, s^R_1), s^R_{2,I}) - V(K^E_1 + k_s(K^E_1, s^R_{1}), s^R_{2,E}) \right]$$
$$+ \delta \left[ C_s(s^R_{2,E}) - C_s(s^R_{2,I}) \right].$$

Comparing this expression with the corresponding profit function $\Pi^{I,O}_{total}$ under open source without commitment we can see that the profit functions for the incumbent are exactly
the same under open source (O) and reciprocal open source (R); only investment levels
are different. Whilst this may seem surprising at first glance, it is actually intuitive to
understand: whilst there is no sharing of knowledge gains with rivals under open source (O),
the incumbent does compensate the customer in period one for the potential knowledge gains
that rivals could obtain from running the first period. These inter-period transfers result in
overall payoffs that are not very different from the outcome under reciprocal open source (R).

3.5 Comparison: Determinants of Open v.s. Closed Source

We can now attempt to understand under which conditions the incumbent will optimally
choose an open source license (O) or (R) or a closed source license (C) to deliver his knowledge-
based service. Hence, we need to compare social welfare and profits of the games \( \Gamma_C, \Gamma_O \) and
\( \Gamma_R \).

Before we solve this problem, let us first understand which license would be optimal to
choose if the incumbent were to sell a commodity good (rather than a service). For a typical
commodity good, there are no knowledge gains, so we assume \( k_s(K, s) = 0 \). Moreover, the
value of a commodity transaction can depend positively on the knowledge \( K \) of the good
producer but there typically is no benefit from any irreversible ex-ante investment on the
customer side; therefore, we assume that \( V(K, s) \) only depends on \( K \), i.e. \( V(K, s) = \bar{V}(K) \).
Comparing between the three available licensing options, we observe that overall surplus is
the same for each of them, but only closed source (C) allows the incumbent to extract all of
it. Hence, we observe:

Observation In the model of ex-ante investment, an incumbent will never choose open
source (O) or reciprocal open source (R) for a commodity good transaction: in terms of
profits, these choices are strictly dominated by closed source (C).

For the provision of knowledge-based services, however, closed source is likely to be sub-
optimal. Before we discuss this scenario in detail we shall, for the sake of clarity, we introduce
the following notation: we denote as \( SW(K, s_1, s_2) \) the social welfare that is produced over
the course two periods if a service firm with an initial knowledge stock of \( K \) runs the project
in both periods and the customer invests amounts of \( s_1 \) and \( s_2 \), respectively:

\[
SW(K, s_1, s_2) \equiv V(K, s_1) - C_s(s_1) + \delta [V(K + k_s(K, s_1), s_2) - C_s(s_2)]
\]

This function has the following important property:

**Lemma 4.** Social welfare \( SW(K, s_1, s_2) \) is strictly concave in \( s_1 \), and it is also strictly concave
in \( s_2 \).

**Proof.** Concavity in both variables follows directly from the fact that \( \frac{\partial^2 C_s}{\partial s_1 \partial s_2} > 0 \) and that \( \frac{\partial^2 V}{\partial K^2}, \frac{\partial^2 V}{\partial s^2} \) and \( \frac{\partial^2 k_s}{\partial s^2} \) are all negative.
We can now rank the social welfare that is generated under each of the three games of ex-ante investment:

**Proposition 5.** If open source (O) does not lead to excessive investment in period one (i.e., \( s_1^O \leq s_1^* \)), social welfare under open source (O) is strictly higher than under closed source (C). Even if open source leads to excessive investment in period one, social welfare under open source is higher than under closed source as long as \( SW(K_1^I, s_1^O, 0) > SW(K_1^I, 0, 0) \).

**Proof.** The result is a direct consequence of the concavity of \( SW(K, s_1, s_2) \) in \( s_1 \) and \( s_2 \): whenever \( s_1^* > s_1^O > s_1^C = 0 \) we can use the fact that \( s_2^* > s_2^O, s_2^C \) together with concavity to conclude that

\[
SW(K_1^I, s_1^*, s_2^*) > SW(K_1^I, s_1^O, s_2^O) > \ldots > SW(K_1^I, s_1^C, 0) = SW(K_1^I, s_2^C).
\]

Note that the second last inequality relies only on the concavity of \( SW(K, s_1, s_2) \) in \( s_2 \) and remains true independently of whether \( s_1^O \) is excessive or not. \( \square \)

By the same argument, we also have

**Corollary 5.1.** If reciprocal open source (R) does not lead to excessive investment in period one (i.e., \( s_1^R \leq s_1^* \)), social welfare under reciprocal open source (R) is strictly higher than under open source (O). Even if reciprocal open source leads to excessive investment in period one, \( SW(K_1^I, s_1^R, s_2^O) > SW(K_1^I, s_1^O, s_2^O) \) is a sufficient condition for social welfare under reciprocal open source (R) to be higher than under open source (O). Then, if ranked ("\( \gtrsim \)"") by social welfare, we have that \( R \gtrsim O \gtrsim C \).

Even though with open source licenses the incumbent must surrender a fraction of the total surplus to the customer, the gain in overall welfare (as compared to closed source) can be sufficiently large that the incumbent’s profits are higher under open source (O) or reciprocal open source (R) than under closed source: to see this, note that we can rewrite the previously derived profit functions under closed source, open source and reciprocal open source as

\[
\Pi_{total}^{I,C} = SW(K_1^I, s_1^C, s_2^C), \\
\Pi_{total}^{I,O} = SW(K_1^I, s_1^O, s_2^O) - SW(K_1^E, s_1^O, s_2^O), \quad \text{and} \\
\Pi_{total}^{I,R} = SW(K_1^I, s_1^R, s_2^R) - SW(K_1^E, s_1^R, s_2^R).
\]

We can thus express the difference in profit between open source (O) and closed source (C) as

\[
\Pi_{total}^{I,O} - \Pi_{total}^{I,C} = [SW(K_1^I, s_1^O, s_2^O) - SW(K_1^I, s_1^C, s_2^C)] - SW(K_1^E, s_1^O, s_2^O).
\]
The first term reflects the surplus gain under open source due to more efficient investment size and has positive sign under the conditions that we have discussed previously. The second term is negative and represents the loss in profit due to competition by entrants. The comparison between reciprocal open source (R) and closed source (C) yields a very similar result. Summarizing our results, we find:

**Observation**  In the model of ex-ante investment, a knowledge-based service firm will choose open source (O) or reciprocal open source (R) rather than closed source (C) whenever the gains in social welfare from more efficient ex-ante investment size outweigh the loss in profit due to the presence of entrants.

4  Knowledge Sharing and Ex-post Collaboration

We now turn to the question how much collaboration effort the customer will provide ex-post (after signing a service contract) under each of the three different licensing regimes \(\{C, O, R\}\). Whenever the service provider cannot commit to share his dynamic knowledge gains, customers need to weigh carefully the benefits versus the downsides of more collaboration (for example, in the form of information disclosure): although more collaboration increases today’s service value, it may also reduce the customer benefits from tomorrow’s services. This is because the increasing knowledge gap between the incumbent and the entrants enables the incumbent to extract more customer surplus in the future. We will see that a firm’s commitment to freely share all dynamic knowledge gains with its rivals (as it is, for example, the case under a reciprocal open source license) can help ameliorate customer concerns about hold-up and induce stronger customer collaboration.

As we have done before, we analyze separate games \(\tilde{\Gamma}_C, \tilde{\Gamma}_O, \tilde{\Gamma}_R\) for each of the three licensing options of closed (C), open (O) and reciprocal (R) open source, taking the general model that we discussed in section 2.3 as the foundation of our setup. Since we are interested in ex-post collaboration levels, we simplify our analysis by assuming that the size of the service project is fixed. Hence, the customer’s ex-ante investment always takes some constant exogenous value \(\bar{s} > 0\). This allows us to define a reduced form of the service value function

\[
V(K, x^i) \equiv V(K, \bar{s}, x^i) - C_s(\bar{s})
\]

and to introduce a more compact notation for the learning function:

\[
k_x(K, x) \equiv k(K, \bar{s}, x)
\]

As before, we will assume that under closed source (C) and open source (O) the dynamic knowledge gains are not shared with competitors whereas they are fully shared under reciprocal open source (R).\(^7\) We also make the same assumptions regarding the initial knowledge

\(^7\)In particular, we assume that if firm \(i\) wins the first period service contract and the customer collaborates
stock of the entrants as before: at the beginning of the first period of the game, knowledge stocks are set to their initial values

\[
K_i^1 = \begin{cases} 
K_i^1 & \text{for the incumbent } i = 1 \\
0 & \text{for all entrants } i \in \{2, 3, \ldots, N\} \text{ in game } \tilde{\Gamma}_C \\
K_i^E & \text{for all entrants } i \in \{2, 3, \ldots, N\} \text{ in games } \tilde{\Gamma}_O, \tilde{\Gamma}_R 
\end{cases}
\]

Subsequently, we play the following game for two periods where each period comprises of the following steps:

1. Each service firm submits a bid \( p_i^t \).
2. The customer selects the service firm \( i \in \{1, \ldots, N\} \) that offers her the highest overall surplus, taking into account how her choice affects her surplus in future periods.
3. The customer determines her level of collaboration \( x_i^t \) with this firm and incurs collaboration cost \( C_x(x_i^t) \).
4. The service value materializes and knowledge stocks of all firms are adjusted according to the corresponding equation of motion.

Again, payoffs originating from the second period will be discounted with a factor of \( \delta > 0 \).

### 4.1 Benchmark: Planner’s Solution

If we were to ask a social planner to choose the actions that maximize overall welfare, the planner would always choose to let the firm with the highest knowledge stock deliver the service. The two first-order conditions that pin down the socially optimal levels of collaboration in both periods, \( x_1^* \) and \( x_2^* \) are

\[
\frac{\partial V}{\partial x} \bigg|_{(K_i^1, x_i^1)} + \delta \frac{\partial V}{\partial K} \bigg|_{(K_i^1 + k_x(K_i^1, x_i^1), x_i^2)} \cdot \frac{\partial k_x}{\partial x} \bigg|_{(K_i^1, x_i^1)} = C_x'(x_2^*) \quad \text{and} \quad (14)
\]

\[
\frac{\partial V}{\partial x} \bigg|_{(K_i^1, x_i^1)} + \delta \frac{\partial V}{\partial K} \bigg|_{(K_i^1 + k_x(K_i^1, x_i^1), x_i^2)} \cdot \frac{\partial k_x}{\partial x} \bigg|_{(K_i^1, x_i^1)} = C_x'(x_1^*) \quad \text{and} \quad (15)
\]

The first of those two equations is of a form that we will continue to incur throughout the rest of our analysis. Thus, we simplify notation by introducing the function \( \bar{x}(K) \) to refer to the \textit{statically optimal collaboration level} that is implicitly defined by the first-order condition

\[
\frac{\partial V}{\partial x} \bigg|_{(K, \bar{x}(K))} = C_x'(\bar{x}(K)).
\]

with this firm with intensity \( x_i^1 \), the second period knowledge stocks will be

\[
K_j^2 = \begin{cases} 
k_i^1 + k_x(K_i^1, x_i^1) & \text{for firm } j = i \\
k_i^1 + \lambda k_x(K_i^1, x_i^1) & \text{for all other firms } j \neq i 
\end{cases}
\]

with \( \lambda = 0 \) for closed source (C) and open source (O), and \( \lambda = 1 \) for reciprocal open source (R).
Since $\frac{\partial^2 V}{\partial x \partial K} > 0$, $\bar{x}(K)$ is strictly increasing in $K$. Using this new notation, we can write the optimality condition for $x_2^*$ as

$$x_2^* = \bar{x}(K_1^l + k_x(K_1^l, x_1))$$

Since the optimality condition for $x_1^*$ comprises of both the static term and a strictly positive dynamic term, we can also bound the socially optimal first collaboration in period one from below:

$$x_1^* > \bar{x}(K_1^l)$$

The economic interpretation of the two optimality conditions for $x_1^*$ and $x_2^*$ is straightforward: optimal collaboration in the second period is equal to the statically optimal level whereas the optimal collaboration level in the first period exceeds the statically optimal level because the planner fully accounts for the dynamic knowledge gains from collaboration.

### 4.2 Game $\Gamma_C$: Closed Source

Under closed source, the incumbent has monopoly power over the customer and thus extracts all customer surplus. However, in sharp contrast to ex-ante investment, the monopolist’s surplus extraction does not completely eliminate the customer’s incentive to collaborate ex post: in the second period, the customer will choose irrespectively of the already paid and sunk service price $p^C_2$ her optimal level $x^C_2$ of ex-post collaboration is simply the statically optimal level,

$$x^C_2 = \bar{x}(K_1^l + k_x(K_1^l, x_1))$$

Anticipating that the customer’s optimal ex-post collaboration choice will be $x^C_2$, the incumbent will set a price for the second period that leaves the customer with exactly zero surplus if she collaborates optimally:

$$p^C_2 = V(K_1^l + k_x(K_1^l, x_1), x^C_2) - C_x(x^C_2)$$

The fact that the second period customer surplus is always zero irrespectively of the incumbent’s knowledge stock implies that the customer’s actions in the first period do not affect her second period surplus. Her first-period collaboration choice $x^C_1$ is thus also governed by the optimality condition of a static model, hence

$$x^C_1 = \bar{x}(K_1^l)$$

We can see that the customer’s collaboration in the first period is inefficiently low from a social point of view: this occurs because she does not participate in the benefits of the incumbent’s
dynamic knowledge gains. Moreover, as $x_1^C < x_1^*$, the incumbent’s second period knowledge stock $K_2^I = K_1^I + k_x(K_1^I, x_1^*)$ will fall short of the socially optimal amount $K_1^I + k_x(K_1^I, x_1^*)$, which in turn implies that customer collaboration in the second period will also be inefficiently low.

In summary, we find:

**Proposition 6.** In the game $\tilde{\Gamma}_C$ of ex-post collaboration under closed source, the incumbent firm extracts all surplus. Customer collaboration is inefficiently low in both periods and does not account for dynamic knowledge gains: $x_1^C = \bar{x}(K_1^I)$ and $x_2^C = \bar{x}(K_1^I + k_x(K_1^I, x_1^C))$. Incumbent profits (and total surplus) amount to

$$\Pi_{total}^C = V(K_1^I, x_1^C) - C_x(x_1^C) + \delta [V(K_1^I + k_x(K_1^I, x_1^C), x_2^C) - C_x(x_2^C)].$$

### 4.3 Game $\tilde{\Gamma}_O$: Open Source (without commitment)

As we shall see, the case in which the incumbent discloses his software as open source but does not commit to sharing any dynamic knowledge gains bears substantial similarity to the situation under closed source. In particular, the incumbent’s inability (or unwillingness) to commit not to hold up his customer for knowledge that was obtained from earlier customer collaboration will leave the customer hesitant to collaborate beyond the statically optimal level in first place.

#### 4.3.1 If the incumbent is awarded the first period

We solve again by backward induction, starting with the second period. We will use indexes $2,ij$ to denote second period quantities if firm $i$ serves in the first and firm $j$ serves in the second period. If the incumbent has run the first period project and received a customer collaboration of $x_{1,I}$, the incumbent’s knowledge stock in period two amounts to $K_2^I = K_1^I + k_x(K_1^I, x_{1,I})$. Since the entrants’ knowledge stocks remain constant, the incumbent will charge for the second period project a price of

$$p_{2,II} = V(K_1^I + k_x(K_1^I, x_1), x_{2,II}) - C_x(x_{2,II}) - [V(K_1^E, \bar{x}(K_1^E)) - C_x(\bar{x}(K_1^E))].$$

The customer will choose an optimal collaboration level $x_{2,II}$ of

$$x_{2,II}^O = \bar{x}(K_1^I + k_x(K_1^I, x_1))$$

and will obtain a second period surplus of

$$S_{2,I}^O = V(K_1^E, \bar{x}(K_1^E)) - C_x(\bar{x}(K_1^E)).$$

---

8Note that, from our assumptions regarding the cross-derivatives of $V(K,s,x)$ follows immediately that $V(K,x)$ has the same properties, in particular $\frac{\partial^2 V}{\partial x \partial K} > 0$, which explains this result.
Note that the customer’s period two surplus is independent of the collaboration choice \( x_{1,I} \) in period one. We can thus already conclude that with the incumbent, the customer’s optimal period one collaboration \( x^{O}_{1,I} \) will only reach the statically optimal level \( x^{O}_{1,I} = \bar{x}(K^{I}_{1}) \) and fall short of the efficient benchmark, i.e. \( x^{O}_{1,I} < x^{*}_{1} \).

### 4.3.2 If an entrant is awarded the first period

The problem that first period collaboration incentives are compromised due to lack of commitment on the side of the service firm would be less severe if the customer were to choose an entrant in period one: second period knowledge stocks would amount to \( K^{I}_{2} = K^{I}_{1} \) for the incumbent and \( K^{E}_{2} = K^{E}_{1} + k_{2}(K^{E}_{1}, x_{1,E}) \) for the entrant. Just as in the previous model of ex-ante investment, second period outcomes if the entrant has served period one depend on whether the entrant’s knowledge gain from learning allows him to surpass the incumbent’s knowledge stock (“substantial learning”). We shall again use the indexes \( \{H, L\} \) to refer to the firm with the highest and second highest period two knowledge stocks, respectively:

\[
K^{H}_{2} \equiv \max(K^{I}_{1}, K^{E}_{1} + k_{2}(K^{E}_{1}, x_{1,E})) \quad \text{and} \quad K^{L}_{2} \equiv \min(K^{I}_{1}, K^{E}_{1} + k_{2}(K^{E}_{1}, x_{1,E})).
\]

The most knowledgeable firm \( H \) wins the period two contract and charges a price \( p_{2,EH} = V(K^{E}_{2}, \bar{x}(K^{H}_{2})) - C_{x}(\bar{x}(K^{H}_{2})) - [V(K^{L}_{2}, \bar{x}(K^{L}_{2})) - C_{x}(\bar{x}(K^{L}_{2})]) \), leaving the customer with a second period surplus of

\[
S^{O}_{2,EH} = V(K^{L}_{2}, \bar{x}(K^{L}_{2})) - C_{x}(\bar{x}(K^{L}_{2}))
\]

if the customer collaborates at her optimal level \( x^{O}_{2,EH} = \bar{x}(K^{H}_{2}) \). We can see that just like before, the second period customer surplus \( S^{O}_{2,EH} \) depends on earlier collaboration \( x_{1,E} \) only if learning is non-substantial. If learning is substantial, the customer nevertheless participates in the surplus gain from her first period collaboration: since the entrant will make positive profits in the second period, these profits (which do depend on the first period collaboration level \( x_{1,E} \)) can be pledged when the entrant attempts to win the customer in period one: hence, overall customer surplus for both periods will reflect some benefit arising from dynamic knowledge gains,

\[
S^{O}_{E,total} = V(K^{I}_{1}, x^{O}_{1,E}) - C_{x}(x^{O}_{1,E}) + \delta \left[ V(K^{I}_{1} + k_{2}(K^{I}_{1}, x^{O}_{1,E}), \bar{x}(K^{E}_{2})) - C_{x}(\bar{x}(K^{E}_{2})) \right]
\]

and the customer chooses a first period collaboration level \( x^{O}_{1,E} \) according to the first-order condition\(^9\)

\[
\frac{\partial V}{\partial x} \bigg|_{(K^{I}_{1}, x^{O}_{1,E})} + \delta \frac{\partial V}{\partial K} \bigg|_{(K^{I}_{1} + k_{2}(K^{E}_{1}, x^{O}_{1,E}), \bar{x}(K^{E}_{2}))} \cdot \frac{\partial k_{2}}{\partial x} \bigg|_{(K^{E}_{1}, x^{O}_{1,E})} = C'_{x}(x^{O}_{1,E}).
\]

\(^9\)Note that as a consequence of the envelope theorem in this first-order condition all terms related to changes in \( \bar{x}(K^{E}_{2}) \) as a function of \( x^{O}_{1,E} \) cancel out.
We observe the following: not only does first period collaboration under the entrant account for dynamic knowledge gains, but collaboration might even become socially excessive: this situation can arise because marginal returns to knowledge in the second term on the left-hand side are higher if the knowledge stock is lower, which will occur whenever innovation is non-substantial. On the other hand, marginal returns to collaboration as shown in the first left-hand side term are lower (due to the lower knowledge stock of the entrant) than in the efficient benchmark. Thus, first period collaboration with the entrant may higher or lower than the efficient level, depending on which of the two effects prevails.

However, we can show that the possibility that the entrant may win the first period in equilibrium (and collaboration levels may come close to or even exceed the efficient benchmark) is of purely theoretical nature:

**Proposition 7.** In the game $\tilde{\Gamma}_O$ of collaboration under open source (without commitment) the incumbent can always win both periods, and he always has incentive to do so. The equilibrium collaboration in both periods is the same as under closed source: $x_{1,I}^O = x_{1,I}^C$, and $x_{2,II}^O = x_{2,II}^C$. In particular, collaboration levels are inefficiently low in both periods, and $x_{1,I}^O$ does not account for any dynamic knowledge gains. Finally, social welfare is the same as in the game $\tilde{\Gamma}_C$ of closed source, but the incumbent’s profits are strictly lower than under closed source.

**Proof.** see appendix.

### 4.4 Game $\tilde{\Gamma}_R$: Reciprocal Open Source

If the incumbent commits to sharing his dynamic knowledge gains by using a reciprocal open source license, customer collaboration will be substantially higher than in the previous cases – in fact, it may even exceed the efficient level in both periods.

Because no entrant can ever match the offer of the incumbent when all knowledge gains are fully shared among firms, the incumbent will always win both periods. We therefore show our results only for this case and delegate the rest to the appendix. Let’s first look at the second period. If the incumbent has already served period one, knowledge stocks will be $K_2^I = K_1^I + k_x(K_1^I, x_{1,I}^R)$ for the incumbent and $K_2^E = K_1^E + k_x(K_1^I, x_{1,I}^R)$ for the entrant, which allows for a surplus extraction of

$$
p_{2,II}^R = V(K_1^I + k_x(K_1^I, x_{1,I}^R), \bar{x}(K_2^I)) - C_x(\bar{x}(K_2^I)) - [V(K_1^E + k_x(K_1^I, x_{1,I}^R), \bar{x}(K_2^E)) - C_x(\bar{x}(K_2^E))]
$$

by the incumbent firm. The customer will thus select a collaboration level of $x_{2,II}^R = \bar{x}(K_1^I + k_x(K_1^I, x_{1,I}^R))$ in the second period, and will choose her first period collaboration level $x_{1,I}$ as
to maximize her overall surplus,
\[
\max_{x_{1,I}^R} \quad V(K_{1,I}^1, x_{1,I}^R) - p_{1,I}^{R} - C_{x}(x_{1,I}^R) \\
+ \delta \left[ V(K_{1}^E + k_{x}(K_{1,I}^1, x_{1,I}^R), \bar{x}(K_{2}^E)) - C_{x}(\bar{x}(K_{2}^E)) \right].
\]

Hence, her equilibrium level of collaboration \(x_{1,I}^R\) with the incumbent in the first period is determined by the first-order condition
\[
\left. \frac{\partial V}{\partial x} \right|_{(K_{1,I}^1, x_{1,I}^R)} + \delta \left. \frac{\partial V}{\partial K} \right|_{(K_{1}^E + k_{x}(K_{1,I}^1, x_{1,I}^R), \bar{x}(K_{2}^E))} \cdot \left. \frac{\partial k_{x}}{\partial x} \right|_{(K_{1,I}^1, x_{1,I}^R)} = C_{x}'(x_{1,I}^R).
\]

An inspection of this first-order condition, combined with previous results, unveils the following:

**Proposition 8.** In the game \(\tilde{\Gamma}_R\) of collaboration under reciprocal open source, the incumbent always wins both periods. The equilibrium levels of customer collaboration are strictly higher in both periods than under closed source or under open source without commitment. It is undetermined how equilibrium collaboration \(x_{1,I}^R\) ranks relative to the socially optimal level: it can be inefficiently low, efficient or even excessive from a social point of view. However, in the limit \(K_{1}^E \to K_{1,I}^1\) the socially optimal levels of collaboration are attained in both periods.

**Proof.** see appendix.

### 4.5 Comparison of the three Licenses

We are now in a position to compare the three licensing options (C), (O) and (R) by adding a period zero of the game in which the incumbent selects a licensing model \(l \in \{C,O,R\}\) and subsequently plays the corresponding subgame \(\tilde{\Gamma}_l\).

Before we turn to the main analysis, let us first for a moment consider the incumbent’s optimal licensing strategy for a highly commoditized good that is not customizable to an individual customer’s need. We capture this by assuming that the delivered value is constant and independent of any collaboration effort on the customer side, i.e. \(V(K, x) = \bar{V}(K)\), and that there is no learning about the customer either, i.e. \(k_{x}(K, x) = 0\). Since generated value does not depend on collaboration and there is no learning either, all three licensing options generate the same social welfare. However, only license (C) allocates the entire surplus to the incumbent. We observe:

**Observation** In the model of ex-post collaboration, a firm that produces a non-customizable commodity good (rather than a service) will never choose any of the open-source licenses (O) or (R). The incumbent will always opt for closed source (C) since it allows for the extraction of the entire surplus.

In sharp contrast to this result, closed source (C) may no longer be optimal if the incumbent attempts to sell a knowledge-based service rather than a commodity good: let us return
to our original assumption that $V(K, x)$ is strictly increasing and concave, and that there are positive learning effects $k_x(K, x)$ from customer collaboration. As we have seen, customer collaboration in both periods then remains inefficiently low under closed source (C) because the customer does not receive any of the additional surplus that is generated from dynamic knowledge gains; the situation under open source without commitment (O) is exactly the same, it only differs from closed source by the aspect that the incumbent has to surrender some surplus to the customer. Hence, in the model of ex-post collaboration, the incumbent will never choose open source without commitment (O) because it is always dominated by closed source (C).

However, under certain circumstances it can be optimal for the incumbent to choose a reciprocal open source license (R) instead of a closed source license (C): in order to see this, first note that if reciprocal open source does not induce socially excessive collaboration in period one, social welfare\(^{10}\) under reciprocal open source is guaranteed to be higher than under closed source. The reason is again the same as in the case of ex-ante investment: social welfare is a concave function of customer collaboration; hence, collaboration levels that are closer to the first-best benchmark will generate a higher social welfare. We summarize our findings:

**Observation** In the model of ex-post collaboration, the incumbent will never adopt open source without commitment (O) because it is always dominated by closed source (C). However, the incumbent may find it profitable to adopt reciprocal open source (R) to boost customer collaboration. Whether or not the incumbent finds it profitable to adopt reciprocal open source depends on whether the gains in social welfare from better collaboration are sufficient to compensate for the incumbent’s reduced ability to extract surplus in the presence of the newly created competition.

5 Extensions

In this section, we present two extensions to our model which explore what happens if some of the assumptions that underlie our model are relaxed.

5.1 Endogenous Choice of Shared Knowledge Stock

So far, we have treated the decision of whether to adopt open or closed source as a binary choice and have considered $K_1^E$ as exogenously given. But this falls short of recognizing that many IT services to big clients are provided by the development of an entire portfolio of software programs. The incumbent then has more fine-grained control over the amount of information he wishes to share with rival firms via open source licensing. It therefore appears more appropriate to consider $K_1^E$ an endogenous quantity.

\(^{10}\)We define here social welfare as the sum of service provider profits and customer surplus.
In this section we ask how much knowledge $K_1^E \in [0, K_1^I]$ the incumbent would optimally share with the public by releasing open source software. The game $\Gamma_E$ of endogenous information choice has the following structure:

1. first, the incumbent decides on the quantity $K_1^E \in [0, K_1^I]$ of knowledge stock that is to be shared via open source licensing with all entrants

2. second, the incumbent plays the game $\Gamma_R$ of reciprocal open source, whereby the initial knowledge stock $K_1^E$ of entrants is set to the previously chosen value.

Remember that the incumbent’s profit function can be rewritten in terms of social welfare functions, which allows us to cast the incumbent’s profit maximization problem into the following form:

$$\max_{K_1^E} \Pi_{total}^L(K_1^E) = SW(K_1^I, s_{1,R}^R, s_{2,I}^R) - SW(K_1^E, s_{1,R}^R, s_{2,E}^R).$$

Observing that the induced ex-ante investments $s_{1,R}^R$ and $s_{2,I}^R$ in both periods approach the socially optimal values in the limit $K_1^E \to K_1^I$, we thus immediately find:

**Observation** When $K_1^E$ is chosen endogenously by the incumbent, he selects $K_1^E$ as to maximize the difference between social welfare and the outside option. The chosen entrant knowledge stock $K_1^E$ will therefore always be lower than the socially optimal level $K_1^{E*} = K_1^I$ that would induce the highest social welfare.

### 5.2 Forking, and Competition by Openness

Another important assumption that we have implicitly made in our model was that whenever the incumbent can not commit to share dynamic knowledge gains and chooses the license (O), all entrants will equally lack such commitment power. This assumption would always be fulfilled if the only commitment vehicle for sharing dynamic knowledge gains was the choice of a reciprocal open source license: since the right to re-license a software under more restrictive or liberal open source licensing terms lies exclusively with the copyright holder, open source essentially constrains an entrant to always deliver the software under the same open source license that was chosen by the incumbent.

However, in practice there are other ways by which an entrant can commit to share dynamic knowledge gains even if the incumbent does not impose such commitment in the software’s licensing terms. Whilst an entrant can’t simply apply different licensing terms to an existing open source software, nothing prevents him from creating a fork of the project (i.e., a derivative work that is based on the incumbent’s original source code but which is subsequently developed along a different path than the original project) which gives him full control over the development process of the forked project. In particular, the entrant can

\[\text{For simplicity, we only present the case (R) of reciprocal open source, but similar results would be obtained for the case (O) of open source without commitment.}\]
choose to commit to a “more open” development process than the incumbent: this can, for example, be attained by making all project related communications, code repositories and decision processes public (usually under the roof of an independent non-profit organization) and by building a reputation for accepting relevant third-party contributions on a regular basis.\footnote{For example, this strategy has been used with great success by the relatively young database service firm SkySQL: in the face of growing doubts about Oracle’s commitment to sharing dynamic knowledge gains (especially, security patches) for its MySQL open source database product, SkySQL created a fork of Oracle MySQL called MariaDB which it keeps developing in a more open development process under the roof of the non-profit MariaDB Foundation.}

If these measures are sufficiently credible to convince the customer that the entrant will share all future dynamic knowledge gains whereas the incumbent is unable (or unwilling) to commit to the same level of knowledge sharing, the incumbent may lose the customer to the entrant: we make a minor modification to the game $\tilde{\Gamma}_O$ of ex-post collaboration under open source to demonstrate this effect. Let us assume that the incumbent does not share any dynamic knowledge gains whereas knowledge stocks under the entrant follow the same equation of motion as in the game $\tilde{\Gamma}_R$ of reciprocal open source. If the customer selects the entrant in period one, period two knowledge stocks will be

$$
K_I^2 = K_I^1 + k_x(K_I^1, x_{1,E})
$$

$$
K_E^2 = K_E^1 + k_x(K_E^1, x_{1,E})
$$

which implies that the customer will choose the incumbent in period two and obtain a second period surplus of

$$
S^{RO}_{2,EI} = V(K^E_1 + k_x(K^E_1, x_{1,E}), \bar{x}(K^E_2)) - C_x(\bar{x}(K^E_2)).
$$

In order to win the customer in period one, the incumbent would thus need to match the overall customer surplus if the entrant was chosen in the first period,

$$
S^{RO}_{total, EI} = V(K^E_1, x_{1,E}^R) - C(\bar{x}_{1,E}^R) + \delta[V(K^E_1 + k_x(K^E_1, x_{1,E}^R), \bar{x}(K^E_2)) - C_x(\bar{x}(K^E_2))],
$$

where $x_{1,E}^R$ is the amount of period one customer collaboration with the entrant from game $\tilde{\Gamma}_R$. However, due to his lack of commitment to share dynamic knowledge gains, the incumbent only produces (if chosen for both periods) an overall social welfare of

$$
SW^{RO}_{total, II} = V(K_I^1, \bar{x}(K_I^1)) - C_x(\bar{x}(K_I^1)) + \delta[V(K_I^1 + k_x(K_I^1, \bar{x}(K_I^1)), \bar{x}_2) - C_x(\bar{x}_2)].
$$

where $\bar{x}_2 \equiv \bar{x}(K_I^1 + k_x(K_I^1, \bar{x}(K_I^1)))$ denotes the statically optimal collaboration level in period two.
For sufficiently high entrant knowledge stock \( K_1^E \), the total social welfare that the incumbent can produce will be less than the total surplus that the customer obtains if she chooses the entrant in period one: to see this, observe that as \( K_1^E \rightarrow K_1^I \), the first period collaboration level with the entrant \( x_{1,E}^R \) approaches the socially optimal one. Second period customer surplus \( S_{2,EI}^{RO} \) then converges towards the second period social welfare in the first-best benchmark. Thus, eventually the overall customer surplus \( S_{total,EI}^{RO} \) must be strictly higher than the social welfare \( SW_{I,total}^{RO} \) that can be produced by the incumbent whose period one collaboration is inefficiently low due to lack of commitment.

Therefore, we conclude that for knowledge-based service firms, the commitment to share dynamic knowledge gains (for example, by forking a software project and making it more “open”) can constitute a powerful weapon for market entry, and the incumbent firm may end up having to make the same commitment in order not to lose the market to the entrant.

6 Concluding remarks

We have presented a simple dynamic model of a knowledge-based service industry that focuses on customer participation in the service transaction. We studied two related models: In the model of ex-ante investment size, we found that knowledge sharing through open source and market sharing is a strategy that the dominant firm can employ to boost customer investment. In the model of ex-post customer collaboration, we found that open source will only boost customer collaboration and increase welfare if the service firm can commit to share dynamic knowledge gains, for example by adopting a reciprocal open source license.

Moreover, we observe that an entrant’s commitment to share dynamic knowledge gains by opening up the development process of an open source product can constitute an aggressive entry strategy. The dominant firm may then embrace reciprocal open source for two reasons: either to boost the collaboration level or in order not to lose the competition.

Although our discussion has been concentrated on IT service industry, the line of argumentation presented in our model is far more general and not limited to software industry in any way. Specifically, we think that our theory can, by its focus on knowledge accumulation in a repeated service relationship, also be used to analyze the competitive dynamics between top management consulting firms such as McKinsey and Boston Consulting Group.

We deliberately focused on the case of a single buyer. In the future, it would be interesting to extend our analysis to the case of multiple buyers when knowledge is transferable across different buyers. This would raise coordination issues among the buyers. More generally, our paper is a first step toward to the IO of a knowledge-based service industry and more studies are needed to characterize the characteristics of this industry with respect to the traditional manufacturing industry.
Appendix

Comparing Second Period Customer Surplus

To show that in the game $\Gamma_O$ the customer has always higher period two surplus if she chooses the entrant in period one, we first prove the following lemma:

**Lemma 9.** Let $W(K) \equiv V(K,s_K) - C_s(s_K)$ be the function of social welfare in a static one-period game where investment $s_K$ is determined by the static first-order condition $C'_s(s_K) = \frac{\partial V}{\partial K} \big|_{(K,s)}$. Then, $W(K)$ is monotonically increasing in $K$, i.e. $\frac{d}{dK} W(K) > 0$.

**Proof.** $\frac{d}{dK} W(K) = \frac{\partial V}{\partial K} \big|_{(K,s)} + \left[ \frac{\partial V}{\partial s} \big|_{(K,s)} - C'_s(s_K) \right] \cdot \frac{ds}{dK} = \frac{\partial V}{\partial s} \big|_{(K,s)}$. □

This simple result, which is a direct consequence of the envelope theorem, enables us to now compare the customer’s second period surplus if she has chosen the entrant in the first period to the surplus that accrues to her if she chooses the incumbent:

$$\Delta S = S_{2,E} - S_{2,I}$$

$$= V(K^L_2,s^O_{2,E}) - C_s(s^O_{2,E}) - \left[ V(K^E_1,s^O_{2,I}) - C_s(s^O_{2,I}) \right]$$

$$= W(K^L_2) - W(K^E_1)$$

Remembering the definition of $K^L_2$, we have that $K^L_1 > K^E_1$, and thus the difference in customer surplus is positive.

**Proof of proposition 2:**

The proposition follows directly from inspecting the first-order equation (13) and comparing it to the efficient benchmark: First, note that the induced investment $s^O_1$ is strictly positive and thus clearly larger than $s^E_1$. Compared to the first order equation (9) which determines the efficient benchmark level $s^*_1$, we see that the first term in equation (13), $\frac{\partial V}{\partial s} \big|_{(K^E_1,s^O_1)}$ is smaller than its counterpart in the efficient benchmark because $K^E_1 < K^I_1$, and $\frac{\partial V}{\partial K \partial s} > 0$. Hence, for sufficiently small $\delta$ investment will be inefficiently low. The second term (i.e., the expression within the square bracket) is ambiguous: on the one hand, we had assumed diminishing marginal returns to knowledge which means that marginal returns to knowledge stock are higher for the entrant than for the incumbent, and thus $\frac{\partial V}{\partial K} \big|_{(K^E_1+k_s(K^E_1,s^O_{2,E}),s^O_{2,E})}$ can be larger with open source than under the benchmark. On the other hand, knowledge stock $K^E_1 + k_s(K^E_1,s^O_1)$ under the entrant is lower than the efficient level, and so is $s^O_{2,E}$. This diminishes both of the two factors within the square bracket relative to the efficient benchmark. Depending on which effect prevails, $s^O_1$ will be inefficiently low, efficient or even excessive.
Full results for the game $\Gamma_R$:

In this section, we provide all results for the game of ex-ante investment under reciprocal open source that were not shown in the main text.

Second period if an entrant has served the first period:

We solve backwards and start our analysis by assuming that an entrant has served the first period when the customer’s ex-ante investment was $s_1$. The resulting knowledge stocks in period two are

$$K_2^I = K_1^I + k_s(K_1^E, s_1)$$
$$K_2^E = K_1^E + k_s(K_1^E, s_1)$$

Clearly, the incumbent has a higher second period knowledge stock than the entrant and will thus win period two. The entrant can at most offer a customer surplus of $V(K_2^E, s_{2,E}^R) - C_s(s_{2,E}^R)$ which means that the incumbent charges a price of

$$p_{2,E}^R = V(K_1^I + k_s(K_1^E, s_1), s_{2,E}^R) - V(K_1^E + k_s(K_1^E, s_1), s_{2,E}^R)$$

and leaves the customer with a period two surplus of

$$S_{2,E}^R = V(K_1^E + k_s(K_1^E, s_1), s_{2,E}^R) - C_s(s_{2,E}^R).$$

Hence, if an entrant has served the first period the customer will optimally choose an ex-ante investment $s_{2,E}^R$ in period two that satisfies the first-order condition

$$\frac{\partial V}{\partial s} \bigg|_{(K_1^E + k_s(K_1^E, s_1), s_{2,E}^R)} = C_s'(s_{2,E}^R).$$

Second period if the incumbent has served the first period:

If, on the other hand, the incumbent has served the first period (which will always happen in equilibrium), second period knowledge stocks are higher than if the client had chosen the entrant in period one:

$$K_2^I = K_1^I + k_s(K_1^I, s_1)$$
$$K_2^E = K_1^E + k_s(K_1^I, s_1)$$

Thus, the incumbent charges a price of

$$p_{2,I}^R = V(K_1^I + k_s(K_1^I, s_1), s_{2,I}^R) - V(K_1^E + k_s(K_1^I, s_1), s_{2,I}^R)$$
and leaves the customer with a period two surplus of
\[ S_{2,t}^R = \mathcal{V}(K_1^E + k_s(K_1^I, s_{2,t}^R), s_{2,t}^R) - C_s(s_{2,t}^R) \]

which is strictly greater than the surplus the incumbent could have offered in period two if the entrant had run the first period project. As a consequence, there will be higher ex-ante investment in period two: the customer will choose an amount \( s_{2,t}^R \) that is determined by the first-order condition
\[
\left. \frac{\partial \mathcal{V}}{\partial s} \right|_{(K_1^F + k_s(K_1^I, s_{2,t}^R), s_{2,t}^R)} = C'_s(s_{2,t}^R).
\]

**First Period:**

In period one, the incumbent can charge a price that extracts the difference to the outside option in the same period plus the additional surplus which the customer receives in the second period if she selects the incumbent in period one: If the customer chooses the incumbent in period one, her overall surplus is
\[ S_{total,I}^R = \mathcal{V}(K_1^I, s_1) - p_{1,I} - C_s(s_1) + \delta S_{2,I}^R \]

whereas an entrant could offer at most to provide the first round service for free, resulting in a customer surplus of
\[ S_{total,E}^R = \mathcal{V}(K_1^E, s_1) - C_s(s_1) + \delta S_{2,E}^R. \]

Thus, the incumbent can charge a first period price of
\[ p_{1,I}^R = \mathcal{V}(K_1^I, s_1) - \mathcal{V}(K_1^E, s_1) + \delta \left[ S_{2,I}^R - S_{2,E}^R \right] \]

and leave the customer with an overall surplus of
\[ S_{total,I}^R = \mathcal{V}(K_1^E, s_1) - C_s(s_1) + \delta \left[ \mathcal{V}(K_1^E + k_s(K_1^E, s_{2,E}^R), s_{2,E}^R) - C_s(s_{2,E}^R) \right]. \]

which implies that optimal first period ex-ante investment \( s_1^R \) will satisfy the first-order condition
\[
\left. \frac{\partial \mathcal{V}}{\partial s} \right|_{(K_1^F, s_1^R)} + \delta \left. \frac{\partial \mathcal{V}}{\partial K} \right|_{(K_1^F + k_s(K_1^F, s_1^R), s_{2,E}^R)} \cdot \left. \frac{\partial k_s}{\partial s} \right|_{(K_1^F, s_1^R)} = C'_s(s_1^R). \]
The incumbent obtains overall profits of

\[
\Pi_{\text{total}}^{I,R} = V(K_1^I, s_1^R) - V(K_1^E, s_1^E) \\
+ \delta [V(K_1^I + k_x(K_1^I, s_1^R), s_2^R) - V(K_1^E + k_x(K_1^E, s_1^E), s_2^E)] \\
+ \delta \left[ C_s(s_2^R) - C_s(s_2^E) \right].
\]

**Proof of proposition 7:**

We will only show that the incumbent will never choose to yield the first period to an entrant because the fact that first period collaboration with the incumbent \(x_{1,I}^O\) remains at the inefficiently low static level was already shown in the main text. For the case in which learning is substantial, it is immediate that the incumbent will never yield the first period. We thus restrict ourselves to analyzing the case in which learning is non-substantial.

If the incumbent yields the first period to an entrant, the incumbent’s overall profits are

\[
\Pi_{\text{total},g2}^O = \delta [V(K_1^I, \bar{x}(K_1^I)) - C_x(\bar{x}(K_1^I))] \\
- \delta [V(K_1^E + k_x(K_1^E, x_{1,E}), \bar{x}(K_2^E)) - C_x(\bar{x}(K_2^E))]
\]

If, on the other hand, the incumbent serves the first period, his profits are

\[
\Pi_{\text{total},p1,p1}^O = p_1^I + \delta [V(K_1^I + k_x(K_1^I, x_{1,I}), x_{2,I}) - C_x(x_{2,I})] \\
- \delta [V(K_1^E, \bar{x}(K_1^E)) - C_x(\bar{x}(K_1^E))]
\]

so the incumbent will be indifferent between winning or losing the first period if he can win the first period at a price of

\[
p_{1,\text{indiff}}^I = \delta [V(K_1^I, \bar{x}(K_1^I)) - C_x(\bar{x}(K_1^I))] \\
- \delta [V(K_1^E + k_x(K_1^E, x_{1,E}), \bar{x}(K_2^E)) - C_x(\bar{x}(K_2^E))] \\
- \delta [V(K_1^I + k_x(K_1^I, x_{1,I}), x_{2,I}) - C_x(x_{2,I})] \\
+ \delta [V(K_1^E, \bar{x}(K_1^E)) - C_x(\bar{x}(K_1^E))]
\]

Note that the sum of the first and the third line gives a negative overall contribution to \(p_{1,\text{indiff}}^I\). Hence, we know that a price \(\hat{p}_1^I\) that results from elimination of these two lines is strictly higher and the incumbent thus prefers to win the first period at this price,

\[
\hat{p}_1^I = -\delta [V(K_1^E + k_x(K_1^E, x_{1,E}), \bar{x}(K_2^E)) - C_x(\bar{x}(K_2^E))] \\
+ \delta [V(K_1^E, \bar{x}(K_1^E)) - C_x(\bar{x}(K_1^E))]
\]
At this price, the customer would enjoy a total surplus of

\[ S_I^{\text{tot}} = V(K_1^I, x_{1,I}^O) - C_x(x_{1,I}^O) \]

\[ + \delta [V(K_1^E + k_x(K_1^E, x_{1,E}^O), \bar{x}(K_2^E)) - C_x(\bar{x}(K_2^E))] \]

if she chooses the incumbent. Since the entrant never wins the second period, he can offer at most a first period price of zero, which would leave a customer surplus

\[ S_E^{\text{tot}} = V(K_E^1, x_{1,E}^O) - C_x(x_{1,E}^O) \]

\[ + \delta [V(K_1^E + k_x(K_1^E, x_{1,E}^O), \bar{x}(K_2^E)) - C_x(\bar{x}(K_2^E))] . \]

Since \( V(K_1^I, x_{1,I}^O) - C_x(x_{1,I}^O) \) is always greater than \( V(K_E^1, x_{1,E}^O) - C_x(x_{1,E}^O) \), it follows that the customer will never choose the entrant at this price \( \tilde{p}_1^I \). Since this price is already higher than the lowest price that satisfies the incumbent’s participation constraint, it is clear that the incumbent will always choose to win the first period.

**Proof of proposition 8:**

Observe that, with the exception of the second term on the left-hand side, the first-order condition (16) is exactly the same as the first-order condition for the socially optimal level of collaboration, eq. (15). The second term deviates from the first-best condition only by the partial derivative \( \frac{\partial V}{\partial K} \) which is evaluated for different values of \( K \) and \( x \): the argument in \( K \) is lower than in the benchmark (it is \( K = K_1^E + k(K_1^I, x_1) \) instead of \( K = K_1^I + k_x(K_1^I, x_1) \)), which, keeping \( x \) constant, would imply that the term involving \( \frac{\partial V}{\partial K} \) should be greater than its benchmark counterpart since we had assumed that \( \frac{\partial V}{\partial K} < 0 \). However, \( x \) is not the same as in the benchmark either (\( \frac{\partial V}{\partial K} \) is evaluated \( x = \bar{x}(K_2^E) \) instead of the larger \( x = \bar{x}(K_2^I) \)). This effect thus goes in the opposite direction and, absent any change in \( K \), would diminish \( \frac{\partial V}{\partial K} \) relative to the benchmark. Depending on which of the two effects prevails, the first period collaboration will be excessive, efficient or inefficiently low.

Finally, if period one collaboration is excessive, so is the collaboration level in period two: the excessive collaboration fosters more learning than under the first-best, i.e. \( k_x(K_1^I, x_{1,I}^R) > k_x(K_1^I, x_1^I) \). Hence, the incumbent’s second period knowledge stock is greater than in the planner’s benchmark. Since we had assumed that \( \frac{\partial^2 V}{\partial K \partial x} > 0 \), the higher knowledge stock then induces a higher (and thus socially excessive) level of collaboration in the second period. The same argument can also be applied to see that inefficiently low collaboration in period one will trigger inefficiently low collaboration in period two as well.

**Game \( \bar{\Gamma}_R \): complete derivation**

In this section we present the complete discussion of game \( \bar{\Gamma}_R \). In order to know the price which the incumbent will charge in period one, we first need to find the customer surplus
that the entrant can offer in both periods:

0.1 If the entrant was awarded period one:

Assume the entrant was awarded the first period, and the customer has collaborated with the entrant with intensity $x_1$. Then, the knowledge stocks at the beginning of period two are

$$K^I_{2,E} = K^I_1 + k_x(K^E_1, x^R_{1,E}),$$

$$K^E_{2,E} = K^E_1 + k_x(K^E_1, x^R_{1,E}).$$

We immediately see that the incumbent will have a higher knowledge stock and win the second period, producing a service value of $V(K^I_{2,E}, \bar{x}(K^I_{2,E}))$ and charging a price of

$$p^R_{2,E} = V(K^I_{2,E}, \bar{x}(K^I_{2,E})) - C_x(\bar{x}(K^I_{2,E}))$$

which leaves the customer with a second period surplus of

$$S^R_{2,E} = V(K^E_{2,E}, \bar{x}(K^E_{2,E})) - C_x(\bar{x}(K^E_{2,E})).$$

Then, the total customer surplus in both periods if the entrant is chosen for the first period is

$$S^R_{total,1E} = V(K^E_{1,E}, x^R_{1,E}) - C_x(x^R_{1,E}) - p^R_{1,E}$$

$$+ \delta \left[ V(K^E_{1,E} + k_x(K^E_1, x^R_{1,E}), \bar{x}(K^E_{2,E})) - C_x(\bar{x}(K^E_{2,E})) \right]$$

which implies that the first-order condition that fixes equilibrium customer collaboration $x^R_{1,E}$ in period one must be

$$\frac{\partial V}{\partial x}(K^E_{1,E}, x^R_{1,E}) + \delta \left. \frac{\partial V}{\partial K} \right|_{(K^E_{1,E} + k_x(K^E_1, x^R_{1,E}), \bar{x}(K^E_{2,E}))} \left. \frac{\partial k_x}{\partial x} \right|_{(K^E_{1,E}, x^R_{1,E})} = C'_x(x^R_{1,E}).$$

In other words, just like under open source without commitment, customer collaboration with the entrant can be excessive or inefficiently low. In any case, since the entrant can not win period two, the best offer he can make is a price of zero, so customer surplus if she goes with the entrant in period one is equal to

$$S^R_{total,E} = V(K^E_{1,E}, x^R_{1,E}) - C_x(x^R_{1,E})$$

$$+ \delta \left[ V(K^E_{1,E} + k_x(K^E_1, x^R_{1,E}), \bar{x}(K^E_{2,E})) - C_x(\bar{x}(K^E_{2,E})) \right].$$

In particular, note that the second period surplus is lower if the entrant has run the first period than if the incumbent has done so; this arises because the entrant realizes weaker
learning effects due to his lower knowledge stock. The incumbent will fully extract this additional second period surplus that he generates in case he wins the first period by charging a proportionately higher price in period one:

**If the incumbent was awarded the first period:**

As we have discussed in the main text, if the customer chooses the incumbent in period one, knowledge stocks evolve as

\[
K_{2,I}^I = K_1^I + k_x(K_1^I, x_{1,E}^R), \\
K_{2,I}^E = K_1^E + k_x(K_1^I, x_{1,E}^R).
\]

and the customer will obtain a second period surplus of

\[
S_{2,II}^R = V(K_1^E + k_x(K_1^I, x_{1,I}^R), \bar{x}(K_{2,I}^E)) - C_x(\bar{x}(K_{2,I}^E)),
\]

and her overall surplus as a function of the price \(p_{1,I}^R\) charged by the incumbent in period one will be given by

\[
S_{total, I}^R = V(K_1^I, x_{1,I}^R) - C_x(x_{1,I}^R) - p_{1,I}^R + \delta \left[ V(K_1^E + k_x(K_1^I, x_{1,I}^R), \bar{x}(K_{2,I}^E)) - C_x(\bar{x}(K_{2,I}^E)) \right].
\]

The incumbent will charge a price that gives the customer exactly the same surplus as if she chooses the entrant in period one so that \(S_{total, I}^R = S_{total, E}^R\). Solving for \(p_{1,I}^R\), we obtain

\[
p_{1,I}^R = V(K_1^I, x_{1,I}^R) - C_x(x_{1,I}^R) - \left[ V(K_1^E, x_{1,E}^R) - C_x(x_{1,E}^R) \right] + \delta \left[ S_{2,II}^R - S_{2,EI}^R \right].
\]

As mentioned before, one can see that the higher second period surplus under the incumbent is fully extracted via the first period price. The customer is left with an overall surplus of

\[
S_{total, I}^R = V(K_1^I, x_{1,I}^R) - C_x(x_{1,I}^R) + \delta \left[ V(K_1^E + k_x(K_1^I, x_{1,I}^R), \bar{x}(K_{2,I}^E)) - C_x(\bar{x}(K_{2,I}^E)) \right]
\]

whereas the incumbent realizes a total profit of

\[
\Pi_{total, I}^R = V(K_1^I, x_{1,I}^R) - C_x(x_{1,I}^R) - \left[ V(K_1^E, x_{1,E}^R) - C_x(x_{1,E}^R) \right] + \delta \left[ V(K_1^E + k_x(K_1^I, x_{1,I}^R), \bar{x}(K_{2,I}^E)) - C_x(\bar{x}(K_{2,I}^E)) \right]
\]

\]
References


